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Abstract: We investigate the feasibility of defining, modelling and projecting of (scaled) mortality improvement rates along cohort years-of-birth; that is, using a cohort perspective. This is in contrast to the approach in the literature which has considered mortality improvement rates that are defined by reference to changes in mortality rates over successive calendar years, that is, using a period perspective. In this paper, we offer a comparison of the 2 parallel approaches to modelling and forecasting using mortality improvement rates. Comparisons of simulated life expectancy and annuity value predictions (mainly by the cohort method) using the England & Wales population mortality experiences for males and females under a variety of controlled data trimming exercises are presented and comparisons are also made between the parallel cohort and period based approaches.

Modelling and projecting mortality improvement rates using a cohort perspective

1. Introduction

The Lee-Carter model has become widely used for the modelling and forecasting of age specific mortality rates. In its standard formulation, the model introduced by Lee and Carter (1992) is as follows:

$$\log m_{xt} = \alpha_x + \beta_x \kappa_t$$

The model is fitted to the observed data and then a projection is based on the extrapolation of the time index κ_t using standard ARIMA time series methods. A random walk with drift has been the most commonly used model for describing the past trends in κ_t ; and the implication of the assumption that a linear trend will continue in the future is that mortality rates at all ages will follow a path of exponential decline over time. As noted by several commentators, this assumption of age invariance in the rate of decline has not been borne out by the recent experience of several countries, with mortality improvement rates reducing over time at the youngest ages and increasing over time at the oldest ages (Lee and Miller, 2001; Booth et al, 2002; Renshaw and Haberman, 2003; Bongaarts, 2005).

In the modelling of mortality dynamics, it is well known that there are advantages if the underlying stochastic process that generates the time index can be assumed to be invariant over time. One of the standard ways in time series analysis of transforming a non-stationary series into a stationary one is by taking differences in the data or "de-trending" (see, for example, Li et al, 2011; Mitchell et al, 2011). The approach of this paper is to use the concept of differencing in the context of defining and modelling cohort-based mortality improvement rates, as a follow up to the consideration of period-based in mortality improvement rates in Haberman and Renshaw (2012)

The starting point for the approach to modelling in this paper is the (Poisson) log-bilinear formulation of the *LC* (Lee and Carter, 1992) parametric age-period model by Brouhns *et al.* (2002), which is used as a means of modelling and then projecting mortality rates (a structure that we refer to as *MR*). Based on the partial derivative of this structure with respect to period, Haberman and Renshaw (2012) describe a parallel alternative approach based on the modelling and projection of mortality improvement rates (which we refer to as *MIR*), which can then be converted back to into *MR* projections. The concept of dual parametric structures connecting the two different approaches is also introduced. In this paper, we investigate the feasibility of taking these ideas further by focusing on the partial derivative of the log-bilinear age-period *LC* parametric predictor structure with respect to cohort year-of-birth.

The paper is arranged as follows. In Section 2, we describe the detail of the new approach, which, in the spirit of Haberman and Renshaw (2012), can be classified as a different type of *Route II* approach. In Section 3, we report in detail on the application of this approach to the England & Wales 1961-2007 mortality experiences. In addition, we include a comparison of life expectancy and annuity value predictions with comparable predictions using the *MIR* approach of Haberman and Renshaw (2012). We conclude with a discussion in Section 4.

2. Methodology

2.1 Nomenclature

In Haberman and Renshaw (2012), the modelling of *MIR* as an indirect means of capturing and then projecting *MR* patterns in time, is referred to as the *Route II* approach. This contrasts with the more conventional direct capture and projection of *MR* patterns in time, which we refer to as the *Route I* approach. In this paper, we refine the *Route II* approach by distinguishing between two different types of *MIR*: a) those computed by using a period perspective (*MIRPO*), as described in Haberman and Renshaw (2012) and referred to previously simply as *MIR*; and b) those computed by using a year-of-birth or cohort perspective (*MIRCO*) and these are introduced and investigated here. We are again interested in the application of parametric structures.

2.2 Mortality improvement rates by cohort orientation *MIRCO*

Consider a rectangular mortality data array, arranged into unit square cells of size one year denoted by

$$\left(d_{xt}, e_{xt}, \omega_{xt}\right): \text{ age } x = x_1, x_2, ..., x_k, \text{ period } t = t_1, t_2, ..., t_n$$

where

d_{xt} - reported number of deaths

e_{xt} - matching central exposure to the risk of death

ω_{xt} - prior weights (0/1) indicating empty or omitted cells

and denote

$m_{x,t}$ - central rate of mortality.

We work with the cohort incremental mortality differences $\left(\hat{m}_{x-1,t-1} - \hat{m}_{x,t}\right)$ scaled by dividing by the average of the two adjacent rates, thus

$$\tilde{z}_{xt} = 2\frac{\left(1 - \hat{m}_{x,t}/\hat{m}_{x-1,t-1}\right)}{\left(1 + \hat{m}_{x,t}/\hat{m}_{x-1,t-1}\right)},\ \hat{m}_{x,t} = \frac{d_{xt}}{e_{xt}},\ \tilde{z}_{xt_1} = \omega_{xt_1} = 0,\ \tilde{z}_{x_1 t} = \omega_{x_1 t} = 0. \qquad (1)$$

A perspective on the nature of such (scaled) differences is obtained by considering the crude set of mortality rates in year *t*-1 and recalling that the consecutive differences $\left(\hat{m}_{x-1,t-1} - \hat{m}_{x,t-1}\right)$ by age are predominantly negative, with the exception of ages in the region of the 'accident-hump'. If we now replace $\hat{m}_{x,t-1}$ in the difference with its immediate updated value $\hat{m}_{x,t}$ (although, in an improving mortality environment, we would expect this difference to narrow), we might also still expect the differences to continue to remain negative, otherwise mortality rates in the population would be improving at a notable rate, with the time trend outweighing the ageing effect. Hence,

the *MIRCO* statistics $\tilde{z}_{xt}$ might be expected to be predominately negative under this scenario.

2.3 Model fitting and diagnostics

We proceed to model $\tilde{z}_{xt}$ as the realisations of independent Gaussian random variables $\tilde{Z}_{xt}$ with variable dispersion. Thus $\tilde{Z}_{xt} \sim N\left(\eta_{xt}, \phi_{xt}\sigma^2\right)$ for which

$$E\left(\tilde{Z}_{xt}\right) = \eta_{xt},\ Var\left(\tilde{Z}_{xt}\right) = \frac{\phi_{xt}\sigma^2}{\omega_{xt}}$$

with parametric predictor η_{xt}, prior indicator weights ω_{xt}, scale parameter σ^2 and parametric dispersion structure $\phi_{xt} = \exp\left(\varsigma_x\right)$, which is modelled as a function of age *x*.

The method of fitting, by alternating between the two stages of a joint model fitting procedure and optimising the respective stage deviances, is as described in Section 2.2 of Haberman and Renshaw (2012). Similarly, the follow-up diagnostic checks on the appropriateness or otherwise of the chosen parametric structures and modelling assumptions are as described in Section 2.4 of Haberman and Renshaw (2012).

2.4 Model structure

We focus on the following first moment predictor structure

$$LC: \eta_{xt} = \beta_x \kappa_t,\ \sum_x \beta_x = 1, \tag{2}$$

which is fitted in conjunction with the age specific parametric dispersion structure defined above.

This structure relates back to the (*Route I*) Poisson log-bilinear regression approach to projecting life tables (Brouhns *et al.* (2002)) using the *LC* (Lee and Carter (1992)) parameterised predictor structure connected via a log link function to the central rate of mortality

$$LC: \log m_{xt} = \alpha_x + \beta_x \kappa_t .$$

Then, in continuous time, with $s = t - x$

$$\frac{1}{m_{xt}} \frac{\partial m_{xt}}{\partial s} = \eta_{xt} \tag{3}$$

where η_{xt} is given by (2) subject to the parameter redefinition $\partial\kappa_t/\partial s = \partial\kappa_t/\partial t \mapsto \kappa_t$. Further, $\tilde{z}_{xt}$ as given by (1), may be interpreted as an approximate discrete time analogue of the LHS of (3). Hence, in parallel with the *MIRPO* approach to modelling described in Section 2.3 of Haberman and Renshaw (2012), we have identified a dual structure and procedure for *MIRCO* modelling and the more conventional *MR* modelling.

2.5 Model dynamics and forecasting

The situation regarding the choice of time series model applied to $\hat{\kappa}_t$ in order to generate forecasts κ_{t_n+j} is as described in Section 2.5 of Haberman and Renshaw (2012). We note that, under *MIR* modelling, κ_t is the equivalent of the derivative of κ_t under *MR* modelling. Then, if we assume that typically $\hat{\kappa}_t$ is an $ARIMA(p,1,q)$ time series process under *MR* modelling, we would expect $\hat{\kappa}_t$ to be an $ARMA(p,q)$ time series process under *MIR* modelling. Thus, we focus here on the application of *ARMA* processes. In particular, we shall have occasion to refer to the *AR*(2) process, for which

$$\kappa_t = c + \varphi_1\kappa_{t-1} + \varphi_2\kappa_{t-2} + \varepsilon_t;\ t = t_4, t_5, ..., t_n;\ \varepsilon_t \sim N\left(0, \tau^2\right)\ i.i.d.$$

2.6 Converting to *MR* forecasts

We use

$$m_{x+j,t_n+j} = m_{x+j-1,t_n+j-1}\frac{\left(2-\tilde{z}_{x+j,t_n+j}\right)}{\left(2+\tilde{z}_{x+j,t_n+j}\right)},\ j = 1,2,3,...$$

based on (1), requiring 'starter' mortality rates m_{x,t_n} to convert the *MIRCO* forecasts $\tilde{z}_{x+j,t_n+j}$ into *MR* forecasts m_{x+j,t_n+j}. We set the 'starter' mortality rates by averaging the observed rates over the last three available cohort cells (while ignoring 'end effects') and by locating the resulting averages in the appropriate penultimate cohort cells. Given the cohort orientation of this re-conversion process the resulting *MR* forecasts are confined to the triangular region bounded by the 'current' period t_n, the upper age x_k, and the outer cohort trajectory $t_n - x_1 - 2$. Once converted, we apply the approximation

$$q_{x+j,t_{n+j}} \approx 1 - \exp\left(-m_{x+j,t_n+j}\right)$$

which is subject to diminishing accuracy in the upper age range.

2.7 Topping out by age

The projected mortality rates

$$q_{x+j,t_n+j} : j = 1,2,...,x_k - x;\ \left(x < x_k\right)$$

are extrapolated further along the age axis up to age $\omega\ \left(> x_k\right)$, before computing the indices of interest along a cohort trajectory. We do this using a conic section in the form of a rectangular hyperbola with appropriate asymptote, thus

$$q_{x_k+j,t_n+x_k-x+j} = a + \frac{b}{\omega - x_k + \rho - j},\ \rho > 0;\ j = 1,2,...,\omega - x_k. \tag{4}$$

The position of the asymptote is determined by setting the value of ρ (we set $\rho = 3$) and the coefficients *a* and *b* are determined by equating with the limiting forecast q_{x_k,t_n+x_k-x} when *j* = 0 and by specifying the details of the ultimate attainable probability of death $q_{\omega,t_n+\omega-x}$ (we set $q_{109,t_n+109-x} = 1$ in general).

This differs from the topping-out formula described in Haberman and Renshaw (2012) and is discussed further in Section 4.2.

2.8 Indices of interest

We consider life expectancy and annuity value predictions, computed along cohort trajectories into the future, located in period $t_n + i : i \geq 0$ and considering individuals aged *x* at that time. Generally *i* = 0, located in the most recent, or current period t_n for which data are available. The life expectancy index is computed as

$$e_x(t_n+i) = \frac{\sum_{j\geq 0} l_{x+j}(t_n+i+j)\left\{1-\frac{1}{2}q_{x+j,t_n+i+j}\right\}}{l_x(t_n+i)};\ \ l_{x+1}(t+1) = (1-q_{x,t})l_x(t)$$

and the discounted annuity value is computed as

$$a_x(t_n+i) = \frac{\sum_{j\geq 1} l_{x+j}(t_n+i+j)v^j}{l_x(t_n+i)} = \sum_{j\geq 1} S_{x,t_n+i}(j)v^j$$

with discount factor *v* and survivor index $S_{x,t_n}(t) : t \geq 0$ which represents the probability of survival from age *x* to age $x+t$ on the basis of the mortality experience in the cohort ages *x* in year t_n.

2.9 Simulations

We use the simulation algorithm described in Section 2.9 of Haberman and Renshaw (2012) to generate predictions and prediction intervals. The algorithm is repeated here for completeness.

Algorithm

For simulation $k = 1,2,...,K$

For $j = 1,2,..,J$

1. sample $\xi_k^{*(j)}$ from $N(0,1)$
2. compute $\kappa^*_{t_n+j|k} = \kappa_{t_n+j} + \sqrt{mse_{t_n+j}}\,\xi_k^{*(j)}$

3. compute $\tilde{z}^{*}_{x+j,t_n+j|k}$
4. compute $m^{*}_{x+j,t_n+j|k}$
5. compute $q^{*}_{x+j,t_n+j|k}$

6. apply topping-out
7. compute indices of interest.

where κ_{t_n+j} and mse_{t_n+j} (step 2) denote the chosen time series process forecast and mean square (prediction) error.

3. England & Wales 1961-2007 mortality

3.1 The data

The data comprise the annual numbers of recorded deaths and matching population sizes exposed to the risk of death, as compiled by the UK Government Actuary's Department (GAD) for the England & Wales male and female mortality experiences. The data are cross-classified by individual calendar year 1961-2007 and age last birthday 0-89. (Data for the 1886 cohort year-of-birth, are zero weighted)

3.2 Exploratory data analysis

For males only, we begin by plotting the *MIRCO* responses $\tilde{z}_{xt}$ against year of observation *t* for a representative sequence of

1) fixed ages: 24, 28, …, 84 (Fig 1a), and
2) fixed cohort years-of-birth 1911, 1915, …, 1971 (Fig 1b).

In Fig 1a we also superimpose the average $\bar{\tilde{z}}_{x.}$ of $\tilde{z}_{xt}$ over *t* for each age *x* (horizontal dotted line). Referring to both figures, we note the following principal features: the predominance of negative values; the strong horizontal trend patterns in both sets of panels, which is a weakening feature for the more recent years-of-birth (Fig 1b); and the variable amount of dispersion about the trend, with a show of greater dispersion at the younger ages (Fig 1a) and in the more recent years-of-birth (Fig 1b).

Similar patterns are observed in the case of the female experience.

3.3 Model fitting and diagnostics

We choose to omit the data for age 0 prior to modelling. As argued by Jarner and Kryger (2011), the nature of infant mortality trends is different from that of the trends at older ages and extra complexity in the modelling would be required for the inclusion of age 0. Jarner and Kryger (2011) note also that infant mortality levels are currently very low in many developed countries and have little impact on summary measures like life expectancy and, of course, they play no role in the determination of annuity values at adult ages.

In the next paragraphs, we report on the results for the modelling of both genders.

For the full period 1961 to 2007 (ages 1-89) only, residual plots are presented in Fig 2a for males and Fig 2b for females. These comprise the standardised residuals plotted respectively against period (upper panel), age (centre panel) and cohort year-of-birth (lower panel). In addition, the distributions of the positive and negative residuals across to the data domain, together with the Gaussian Q-Q and half Gaussian residual plots, for each gender, are displayed in Appendix A. We note the satisfactory nature of all of these plots. We highlight the following features: the broad capture of the age effects, period effects and cohort effects, noting that the latter are not explicitly represented in the predictor structure; the uniformity of dispersion in Figs 2a&b which has been achieved by the use of two stage joint model fitting; the lack of any obvious clustering patterns in the distribution of positive and negative residuals (see Figs A1a&b); and evidence of compliance with the Gaussian modelling assumption with variable dispersion (see Figs A2a&b).

The respective parameter estimates for males and females are displayed in Figs 3a&b and arranged as follows: the age modulating index $\hat{\beta}_x$ in the upper LH panel, the period index $\hat{\kappa}_t$ forms part of the display in the centre RH panel, and the dispersion parameter $\hat{\phi}_{xt} = \hat{\phi}_x$ in the upper RH panel: all three remaining panels in these figures refer to the residual plots associated with the time series process applied to $\hat{\kappa}_t$ which we discuss in the next section.

From Figs 3a&b, we note the following key features: the similarity in shape of the matching panels; the pronounced spike in the β_x parameters, which coincides with the positioning of the characteristic 'accident hump' that is associated with static period life tables; the more pronounced nature of the 'accident hump' spike for males relative to females; the general linear nature of the pattern in the period indices $\hat{\kappa}_t$; and the approximately hyperbolic shape of the dispersion parameter. On this last point, we recall that the reciprocal of these parameters determines the weights applied when estimating the parameters β_x and κ_t which, in turn, determine the first moment properties of the model, so that greater weight is given to the older ages when estimating these parameters.

3.4 Model dynamics

For the full period 1961 to 2007 (ages 1-89) only, we present details of the *AR*(2) process applied to the fitted period indices in order to facilitate model projection. Referring to Section 2.5, details of the fitted process read as follow

	Males	Females
c	-8.355 (0.9260)	-8.604 (0.8787)
ϕ_1	-0.5682 (0.1370)	-0.6845 (0.1324)
ϕ_2	-0.4069 (0.1213)	-0.4064 (0.1176)

AR(2) parameter estimates with (standard errors)

In addition, we depict the resulting forecast and the 95% prediction intervals for this process by augmenting the time series $\hat{\kappa}_t$ in the centre RH panels in Figs 3a&b. In

addition, the centre LH panels depict the associated process residuals, while the lower panels depict the Q-Q and half-Gaussian ordered residual plots, which are testing for the Gaussian modelling assumption.

3.5 Back-testing and life expectancy and annuity predictions

In this section, we conduct a back-testing exercise along the lines of Dowd et al (2010) and Haberman and Renshaw (2011) and focus on life expectancy and annuity predictions. We use 2 approaches as in Haberman and Renshaw (2011). In the first approach, we use the data from historical periods for fitting the model that are extended over time, from 1961-1993 through to 1961-2007, in order to predict the indices of interest for the final year in the sequence. In the second approach, we use the data from historical periods for fitting the model that are reduced over time by deleting the oldest years, from 1961-2007 through to 1975-2007, in order to predict the indices of interest for 2007.

We display life expectancy and 4% annuity value simulated 5%, 50%, 95% quantile predictions for males (Fig 4a) and females (Fig4b) in the two upper bands in the panels of the respective figures (labelled *MIRCO*). In the first of these bands, the evolving biennial dynamic simulated predictions $t_n = 1993(02)07$, subject to front-end data deletions are shown in ascending sequence; thus, we are successively using data for 1961-1993, 1961-1995, …, 1961-2007. In the second of these bands, we show static $t_n = 2007$ simulated predictions, where the data have first been subjected to systematic biennial truncations at the start of the period from 1961 to 1975. These are shown in ascending sequence, thereby using data for the periods 1961-2007, 1963-2007, …, 1975-2007. For both of these sets of results, the index *i* is set to zero in Section 2.8. All individual predictions are based on *K* = 2000 simulations.

For comparison, we also display equivalent predictions in the lower two bands of the panels in Figs 4a&b, using the *MIRPO* approach to modelling and predicting described in Haberman and Renshaw (2012). The computation of these results from the *MIRPO* approach differs in the following respects: the data are restricted to ages 20-89 prior to modelling, the *LC* predictor is augmented by the inclusion of a cohort index to read as

$$H_1 : \eta_{xt} = \beta_x \kappa_t + \iota_{t-x},$$

and the use of an *AR*(1) times series process as opposed to an *AR*(2) process. In other respects, such as the fitting by joint modelling, topping out by age based on equation (4) in Section 2.7 (noting we have re-worked the *MIRPO* approach presented in Haberman and Renshaw (2012) using the hyperbolic approach to topping out rather than the quadratic-based method presented in the previous paper) and the numbers of simulations *K* = 2000, the computations are the same. In addition, Figs 4a&b are augmented by the tabulation of the median predictions in Tables 1a&b. Referring to Figs 4a&b and Tables 1a&b, there are some key features that we would like to highlight. Firstly, the prediction intervals simulated using the $MIRCO - LC$ approach are consistently narrower compared with the matching intervals simulated using the $MIRPO - H_1$ approach. Secondly, an examination of the dynamic predictions (bands 1 & 3 in each panel) shows that the stacking angles of the age related prediction intervals become steeper with decreasing age. This is indicative of a slower rate of mortality improvement (over the period 1993-

07) with decreasing age. This feature is also captured by the increasing trend, as age increases, in the standard deviations reported in the sub-tables included in Tables 1a&b. Finally, an examination of the 2007 static predictions (subject to the systematic early period data deletions) shows that the age specific stacking angles under the $MIRPO-H_1$ approach are almost vertical. This feature should be compared with the tendency, under the $MIRCO-LC$ approach, for these angles to become more inclined, away from the vertical with decreasing age.

3.6 Upper 1 in 200 annuity quantiles

In this section, we make brief reference to the changes in the regulatory capital requirements for European insurance companies that are expected to be introduced as part of the Solvency II project. A motivation for these changes is to develop a more realistic approach to the modelling and measurement of all of the main risks to which insurance companies are exposed – and this includes longevity risk for those companies that transact annuity business. A key element is the Solvency Capital Requirement (SCR). As noted by Borger (2010), the SCR may be intuitively defined at the amount of capital necessary at time *t=0* to cover all losses which may occur in the period up to *t=1* with a probability of least 99.5%. We note that an approximate approach to the calculation of the SCR for longevity risk is also permitted (CEIOPS, 2008): this is determined by the change in the net asset value (i.e. the difference between the market value of the assets and the best estimate of the liabilities) at time *t=0* due to a permanent reduction in the future mortality rates for each age by 25%.

Given this potential interest in the 99.5% quantiles for annuity predictions, we take a simple viewpoint and consider only the situation from the perspective of the current time (rather than looking 1 year forward). In presenting these calculations, we acknowledge that we are merely exploring the properties of the extreme percentiles of the predicted distributions at time *t=0,* rather than exploring the full and realistic implications for the SCR in Solvency II terms.

Thus, we have tabulated the relative dispersion of the upper 1 in 200 quantiles from the respective median predictions

$$k_x(t_n)=\frac{a_x^{99.5\%}(t_n)-a_x^{50\%}(t_n)}{a_x^{50\%}(t_n)}\ \text{, so that}\ a_x^{99.5\%}(t_n)=\{1+k_x(t_n)\}a_x^{50\%}(t_n)$$

for the evolving 1993(02)07 period 4% annuity values, cross-classified by age 40(05)75, modelling approach and gender in Table 2, as t_n moves from 1993 to 2007. As a consequence, by selecting the relevant entries from Tables 1a&b and Table 2, it is possible to calculate the 1 in 200 quantiles. Referring to Table 2, we note the smaller relative dispersion measure using the *MIRCO-LC* approach compared with using the $MIRPO-H_1$ approach, matched for age and period: this is a reflection of the narrower prediction intervals using the former approach. We also note the decreasing trend in the values of the respective measure of relative dispersion as we extend the period, and the increasing trend in the values of the respective measure of relative dispersion as we increase age.

3.7 Survivor probability predictions

In Fig 5, we illustrate predicted survivor probability function for the period $t_n = 2007$ by depicting the simulated 5%, 50%, 95% quantiles for ages 40(05)75 in each of the four panels. The two upper panels related to the *MIRCO-LC* approach for males (LH panel) and females (RH panel) using data for the full 1961-07 period (ages 1-89). Similarly, the two lower panels relate to the $MIRPO - H_1$ approach for males (LH panel) and females (RH panel) using data for the full 1961-07 period (ages 20-89). The results in Fig 5, show that the prediction intervals simulated using the $MIRCO - LC$ approach are narrower when compared with the matching intervals simulated using the $MIRPO - H_1$ (upper panel compared with the matching lower panel). This is a feature that is consistent with the findings reported earlier when comparing life expectancy and annuity predictions intervals. We also note a shift in the collective patterns of the predicted curves in the direction of the respective upper RH corner of the panels for females, compared to males, which is consistent with the lighter mortality associated with the female experience (and is an example of the rectangularisation of the survival curve).

4. Discussion

4.1 Alternative parametric structures

Our results indicate that greater structural parametric complexity cannot be justified in the model of Section 3 on the basis that the proposed model successfully captures all of the three key age, period and cohort effects. We note that the Gaussian *MIRCO* response model uses the *LC* bilinear multiplicative age-period effects and the full age range of the data (except for age 0).

However, it is of interest to note that similar results are obtained by switching to the linear additive age-period effects structure

$$H_2 : \eta_{xt} = \beta_x + \kappa_t, \ \sum_x \beta_x = 1$$

(within the same context of Gaussian *MIRCO* response modelling). To illustrate this, details of the fitted H_2 structure and subsequent $\hat{\kappa}_t$ *AR*(2) time series with forecasts under Gaussian *MIRCO* joint modelling for the England & Wales 1961-2007 (ages 1-89) male and female mortality experiences are displayed in the respective Figs 6a&b. The layout of these figures is identical to Figs 3a&b so that direct comparisons can be made. For each gender, we note the remarkable similarity of matching individual panels in these figures subject to the inversion of the β_x parameter patterns (which arise because of the switch from a multiplicative to additive model) and the change of scale in the κ_t parameter patterns. Since we have observed no material differences in the simulated life expectancy and annuity predictions generated using this structure compared with the *LC* structure we have not included these results when constructing Figs 4a&b. However, we make an exception in Section 4.5 when constructing Fig 12 in order to demonstrate this feature.

4.2 Topping-out by age

Clearly the (subjective) choice of topping-out formula and the setting of the ultimate attainable age ω make a contribution to the predicted mortality rates and the subsequent indices of interest that depend on these choices. In Section 7.2 of Haberman and Renshaw (2012), we have illustrated and discussed the potential magnitude of this effect on life expectancy predictions under different settings of ω (and $q_{\omega,t_n+\omega-x}$) using a quadratic topping-out formula on the log scale, which can be rewritten to read as follows:

$$\log\left(q_{x_k+j,t_n+x_k-x+j}\right) = a + b(j+1) + c(j+1)\, j;\ j = 1, 2, ..., \omega - x_k . \qquad (5)$$

Here the coefficients *a*, *b* and *c* are determined by setting $j = -1, j = 0$ and equating with the appropriate observed log mortality rates, in addition to setting the values of (ω, $q_{\omega,t_n+\omega-x}$).

In the current paper, we have replaced expression (5) with expression (4), comprising the formula for a rectangular hyperbola linked directly to the mortality rate (*viz.* probability of death), rather than the log of the mortality rate. The effect of this change is illustrated in Fig 7 in which we plot the combined projected and top-out mortality rates along cohort trajectories. We focus on the period 2007 for ages 40(05)75, under the joint modelling Gaussian *MIRCO* approach with *LC* structure (using data for the age range 1-89), and use the topping-out formulae (4) and (5) to construct the separate panels on display. Note that, in each display panel, we have added 1 to the mortality rates for each (five year) incremental increase in age for greater clarity.

Referring to Fig 7, we note the concave nature of the curved structure imposed on the run of top-out mortality rates, which is by design, and the general similarity of the two sets of curves using the different formulae. However, the reason for our changing to the hyperbolic formula from the quadratic formula in the current paper concerns the potential lack of flexibility in fitting a quadratic based formula, which we have observed in experiments with fitting (5) to the US mortality experiences (details not included). For these data, when setting the choice of ultimate attainable age $\omega = 109$ (or 119) with $q_{\omega,t_n+\omega-x} = 1$ in the use of (5), we find that, due to this lack of flexibility, the run of extrapolated probabilities also attains the value 1 prior to the specified age ω.

4.3 An insight into prediction error in retrospective study

In the spirit of Section 7.3 of Haberman and Renshaw (2012), we have applied the joint modelling Gaussian *MIRCO* approach with *LC* structure, to the England & Wales male and female mortality experiences, restricted to the period 1961-1982 (ages 1-89), and calculated the 1982 predicted life expectancies and 4% annuity values for individuals aged 65(01)80 by the cohort method (using the same time series models and topping-out strategy). Then, using the actual crude mortality rates for the period 1983-2007, the same calculations are made and the resulting relative errors ((predicted-actual)/actual) in the life expectancies and annuity values have been calculated and plotted against age, in the respective upper panels of Fig 8. We note the consistently smaller relative errors for

females compared with males for each age, with close to zero error for females at a number of ages.

We have also calculated and then averaged the relative errors in the log death rates ((predicted-actual)/actual) by age, period and cohort year-of-birth respectively, using the rectangular region bounded by ages 60-89, period 1983-2007, inclusive of the log death rates used to construct the upper panels in Fig 8. The results are depicted in the middle and lower panels of Fig 8. On comparing like for like, again the mean relative errors are consistently lower for females compared with males.

Fig 8 may also be compared with the results shown in Fig 15 in Haberman and Renshaw (2012).

4.4 Forward predictions

All of the predictions reported in Section 3 are focused on the so-called *current* period t_n which does not exceed 2007 for the data in use. In Fig 9, we consider forward predictions. Thus, we set $i = 0,1,...,7$ in the equations given in Section 2.8 for the key indices and show in Fig. 9 the evolving life expectancy and 4% annuity value 5%, 50%, 95% quantile predictions, computed by the cohort method and presented in ascending sequence, for ages 40(05)75 and for both genders of the England & Wales mortality experience. For comparison, we juxtapose predictions using the Gaussian *MIRCO-LC* approach with predictions using the Gaussian $MIRPO - H_1$ approach (both under joint model fitting), with the first two bands depicting the results for males and the lower two bands the results for females in each of the two panels depicting life expectancy and annuity predictions respectively.

From the results in Fig 9, there are some notable features to which we would draw attention. The prediction intervals using the *MIRCO-LC* approach are narrower compared with the matching intervals obtained using the $MIRPO - H_1$ approach. On comparing like with like, the median point predictions using the *MIRCO-LC* approach are more conservative when compared with the matching median point predictions obtained using the $MIRPO - H_1$ approach. For females (bands 3&4), we note the same direction of the inclination of the stacking angles of the evolving prediction intervals; this feature is associated with improving mortality predictions into the future. For males (bands 1&2), we note that the direction of the inclination of the stacking angles (with the exception of age 40) using the *MIRCO-LC* approach is generally in the opposite direction to the inclination of the stacking angles using the $MIRPO - H_1$ approach. This implies an arresting of the forward life expectancy and annuity predictions using the cohort method of modelling.

In order to explain the above unexpected phenomenon i.e. find a reason for the opposing directions in which the forward projected sequences of life expectancy and annuity value predictions progress (in particular for the male experience), we display the underpinning simulated log mortality rate projections at regularly spaced ages 40(05)85 in Fig 10. Here, the two upper panels refer to the *MIRCO-LC* modelling approach and the two lower panels to the $MIRPO - H_1$ approach, with the results for males in the respective LH panels and those for females in the RH panels. Modelling is by joint

fitting throughout. The age specific profiles within each panel taper by 5 years for each 5 year reduction in age, and the profiles as a whole include the log mortality rates contributing to the construction of Fig 9. Referring to the results in Fig 10, we note the following important features. For females (RH frames), we note the close similarity of the matching age specific profiles under the two approaches, including the replication of the finer variation within each profile. We also note the steady parallel reduction in the predicted age specific log mortality rates. For males (LH frames), the patterns under the two approaches are different. First, we note the marked change in the inclination of the evolving age specific trend patterns, common under both approaches, which is centred on the years-of-birth 1947-48. While under the $MIRPO-H_1$ approach (lower LH panel), the trend patterns continue downwards (subject to a less steep rate of decrease), there is a small reversal in the trend under the *MIRCO-LC* approach. Coincidentally, we recall a similar change, also centred on the years-of-birth 1947-48, in the profile of a cohort index fitted to (an earlier version of) these data and depicted in the lower RH frame of Fig 4 in Renshaw and Haberman (2006).

It is also of interest to investigate and compare equivalent forward predictions computed by the period method and presented in Fig 11. This involves suppressing the variation in the period component indices in the formulae for calculating life expectancies and annuities (Section 2.8) and in the topping-out formula (Section 2.7). Referring to the results in Fig 11, we note the regularity of the age specific patterns coupled with the consistent direction of the stacking angles and the narrower prediction intervals using the *MIRCO-LC* approach when compared with matching intervals using the $MIRPO-H_1$ approach.

By comparing like with like in Fig 9 and Fig 11, it is possible to gauge the effect on the life expectancy and annuity median point predictions of switching from computation by the cohort method to computation by the period method.

4.5 USA 1961-2006 mortality experience

Since the *MIRCO-LC* (and H_2) approach to modelling and mortality rate projecting that we have introduced in this paper is apparently novel, we have repeated the analysis for the USA 1961-2006 male and female mortality experiences (ages 1-89). With respect to model fitting, we observe no material differences in the resulting $\hat{\beta}_x, \hat{\kappa}_t; \hat{\phi}_x$ parameter patterns in comparison with those reported for the England & Wales experiences in Figs 3a&b, (except for a small degree of clustering in the positive and negative residuals, the equivalent of Figs A1a&b). The analysis points towards using an *AR*(1) time series for $\hat{\kappa}_t$. Given the different inclination of the stacking angle of the *MIRCO* forward life expectancy and annuity point predictions for males in Fig 9, we depict, for the USA, evolving life expectancy and 4% annuity value 5%, 50%, 95% quantile predictions in ascending sequence (computed by the cohort method), for ages 40(05)75 in Fig 12. Thus, we set $i = 0,1,...,7$ in the equations given in Section 2.8 for these key indices using a starting year of 2006. Here, we have juxtaposed the *MIRCO-LC* and H_2 predictions, and not the $MIRPO-H_1$ predictions of Fig 9. We have done this in

order to illustrate the degree of agreement between *MIRCO-LC* and H_2 predictions which has not previously been illustrated.

Referring to the results in Fig 12 we note the following features. When comparing like with like, we note the close agreement between the *MIRCO-LC* and H_2 predictions. For females (bands 3&4), we note the consistent direction of the inclination of the age specific stacking angles of the evolving prediction intervals, a feature which is associated with improving mortality predictions into the future. For males (bands 1&2), we note again the reversal in the direction of some of the age specific stacking angles; but we note that this is a less marked feature than that occurring in Fig 9.

4.6 Summary comparison of *LC* log-bilinear and dual structured models

It is instructive to list and contrast certain basic features identified when using the three different approaches to modelling parametric age-period predictor structures, *viz*, the *MR* Poisson response log-bilinear structure model and the two Gaussian *MIRPO* and *MIRCO* response bilinear structured models. While some of the following summary statements apply universally, others are data specific and are based on our analyses of the England & Wales and the USA mortality experiences. Assuming a rectangular age-period data array arranged annually we comment as follows:

- We believe that the full scrutiny and reporting of residual plots is essential to lend credibility to the choice of model structure as a general tenet.
- For projection purposes, we have found it necessary to restrict the choice of *ARIMA* time series processes to model the fitted period index to: (i) first order integrated processes when using the *MR* Poisson response approach, and to (ii) non-integrated *ARMA* processes when using the other two approaches based on improvement rates. This may involve compromising on goodness-of-fit.
- The *MIRCO* approach has been shown to capture all three age, period and cohort main effects, using the full age range of the data, unlike the other two approaches (*MR* and *MIRPO*), both of which fail adequately to capture the main cohort effects, even if the age range is shortened.
- Using the *MIRCO* approach, the projection of mortality rates is restricted to the triangular region bounded by the current period, the outer age limits, and the outer cohort trajectory as defined by the data array. The outer cohort restriction does not apply when using the other two approaches.
- The choice and age span of the topping-out formula does contribute materially to the values of the predicted indices of interest such as life expectancy and annuity values.
- The diminution in the annual evolving life expectancy and annuity predictions, computed by the cohort method, for certain ages of the England & Wales male mortality experience using the *MIRCO* approach, was not anticipated.
- There is some evidence that the prediction intervals obtained under the *MIRCO* modelling approach are narrower than the corresponding prediction intervals obtained under the *MIRPO* approach.

We believe that there is scope for further research in exploring the utility of the 2 approaches, *MIRPO* and *MIRCO,* to modelling mortality improvement rates.

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E&W male mortality experience

Age	*Approach*	*1993*	*1995*	*1997*	*1999*	*2001*	*2003*	*2005*	*2007*
40	*MIRCO-LC*	39.47	39.59	40.07	40.83	40.89	41.14	42.02	42.66
	MIRPO-H1	40.32	40.89	41.65	41.94	42.92	42.72	43.30	44.29
45	*MIRCO-LC*	35.54	35.85	35.99	36.20	37.06	37.77	37.08	37.83
	MIRPO-H1	35.91	36.33	36.56	36.50	37.91	38.38	38.86	39.37
50	*MIRCO-LC*	30.65	30.96	32.24	32.36	32.49	32.29	32.88	33.80
	MIRPO-H1	30.67	31.18	32.78	32.32	33.16	33.11	33.88	34.58
55	*MIRCO-LC*	25.76	26.17	26.72	27.27	28.19	28.56	28.78	29.20
	MIRPO-H1	25.61	26.10	26.91	27.50	28.60	29.00	29.20	29.44
60	*MIRCO-LC*	20.69	21.23	22.10	22.47	23.10	23.63	24.39	25.49
	MIRPO-H1	20.41	21.20	22.13	22.42	23.39	24.05	24.60	25.70
65	*MIRCO-LC*	15.88	16.61	17.35	17.95	18.91	19.31	19.84	20.21
	MIRPO-H1	15.64	16.43	17.33	17.83	18.86	19.44	29.86	20.43
70	*MIRCO-LC*	12.13	12.53	13.10	13.56	14.36	15.08	15.62	16.46
	MIRPO-H1	11.91	12.26	12.99	13.45	14.38	14.90	15.59	16.31
75	*MIRCO-LC*	9.08	9.64	9.76	10.05	10.45	11.07	11.77	12.32
	MIRPO-H1	8.87	9.40	9.63	9.83	10.38	10.79	11.51	12.09

Life expectancy median point predictions

Approach	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*
MIRCO-LC	1.12	0.89	1.01	1.28	1.60	1.56	1.54	1.12
MIRPO-H1	1.31	1.31	1.30	1.48	1.78	1.70	1.58	1.10

Standard deviations, computed across rows (period 1993-07)

Age	*Approach*	*1993*	*1995*	*1997*	*1999*	*2001*	*2003*	*2005*	*2007*
40	*MIRCO-LC*	18.69	18.71	18.82	18.98	18.98	19.04	19.22	19.35
	MIRPO-H1	18.88	18.99	19.14	19.21	19.37	19.33	19.44	19.64
45	*MIRCO-LC*	17.72	17.79	17.83	17.88	18.09	18.26	18.08	18.26
	MIRPO-H1	17.82	17.92	17.97	17.96	18.27	18.39	18.50	18.61
50	*MIRCO-JLC*	16.32	16.41	16.80	16.83	16.86	16.80	16.96	17.22
	MIRPO-H1	16.34	16.48	16.94	16.81	17.02	17.01	17.23	17.42
55	*MIRCO-LC*	14.69	14.84	15.03	15.22	15.53	15.65	15.72	15.85
	MIRPO-H1	14.66	14.82	15.10	15.31	15.65	15.79	15.85	15.92
60	*MIRCO-LC*	12.70	12.92	13.28	13.42	13.67	13.89	14.18	14.60
	MIRPO-H1	12.59	12.92	13.29	13.42	13.78	14.05	14.26	14.67
65	*MIRCO-LC*	10.47	10.83	11.19	11.47	11.91	12.09	12.33	12.50
	MIRPO-H1	10.37	10.75	11.18	11.43	11.89	12.17	12.35	12.60
70	*MIRCO-LC*	8.464	8.687	9.006	9.257	9.688	10.07	10.34	10.77
	MIRPO-H1	8.342	8.549	8.957	9.220	9.708	9.991	10.35	10.71
75	*MIRCO-LC*	6.632	6.980	7.058	7.232	7.480	7.851	8.260	8.579
	MIRPO-H1	6.494	6.840	6.982	7.110	7.447	7.703	8.130	8.470

4% annuity value median point predictions

Approach	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*
MIRCO-LC	0.232	0.214	0.288	0.433	0.635	0.728	0.820	0.671
MIRPO-H1	0.246	0.299	0.358	0.489	0.700	0.793	0.853	0.672

Standard deviations, computed across rows (period 1993-07)

Table 1a. E&W 1961-07 male mortality. Evolving biennial 1993(02)07 life expectancy and 4% annuity median predictions (columns), cross-classified by age 40(05)75 and modelling approach (rows), computation along cohort trajectory. K = 2000 simulations. Topping out constraint $q_{109,t_n+109-x} = 1$.

Gaussian joint modelling *MIRCO-LC* approach, data restricted to ages 1-89.
Gaussian joint modelling *MIRPO-H1* approach, data restricted to ages 20-89.
E&W female mortality experience

Age	*Approach*	*1993*	*1995*	*1997*	*1999*	*2001*	*2003*	*2005*	*2007*
40	*MIRCO-LC*	46.03	46.31	46.51	46.66	47.53	46.95	47.38	47.65
	MIRPO-H1	45.83	46.17	46.93	47.32	48.38	48.22	48.68	48.54
45	*MIRCO-LC*	40.60	40.44	41.19	40.74	41.33	41.43	42.18	42.40
	MIRPO-H1	40.74	40.68	41.46	41.43	42.62	42.65	43.18	43.39
50	*MIRCO-LC*	35.12	35.46	36.33	35.63	35.97	36.34	36.81	37.11
	MIRPO-H1	35.25	35.69	36.86	36.50	37.29	37.37	37.73	37.80
55	*MIRCO-LC*	29.78	29.82	30.47	30.59	31.17	31.53	31.87	32.15
	MIRPO-H1	29.69	20.34	31.04	31.29	32.01	32.52	32.60	32.87
60	*MIRCO-LC*	24.27	24.88	25.12	25.47	26.14	26.60	27.19	27.55
	MIRPO-H1	24.21	25.14	25.78	26.04	26.88	27.45	27.62	28.21
65	*MIRCO-LC*	19.23	19.69	20.38	20.96	21.43	21.68	22.41	22.81
	MIRPO-H1	19.24	19.89	20.65	21.01	22.13	22.29	22.64	23.09
70	*MIRCO-LC*	15.06	15.43	15.74	16.14	16.92	17.43	17.91	18.46
	MIRPO-H1	14.93	15.37	15.75	16.23	17.03	17.44	18.01	18.48
75	*MIRCO-LC*	11.69	12.18	12.19	12.32	12.69	13.02	13.50	14.17
	MIRPO-H1	11.45	12.01	12.00	12.12	12.60	12.81	13.34	13.96

Life expectancy median point predictions

Approach	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*
MIRCO-LC	0.60	0.71	0.68	0.90	1.16	1.26	1.23	0.81
MIRPO-H1	1.11	1.07	0.94	1.15	1.36	1.38	1.28	0.82

Standard deviations, computed across rows (period 1993-07)

Age	*Approach*	*1993*	*1995*	*1997*	*1999*	*2001*	*2003*	*2005*	*2007*
40	*MIRCO-LC*	20.11	20.16	20.20	20.23	20.39	20.28	20.35	20.40
	MIRPO-H1	20.07	20.11	20.24	20.33	20.49	20.47	20.55	20.53
45	*MIRCO-LC*	19.03	18.99	19.16	19.06	19.19	19.21	19.38	19.42
	MIRPO-H1	19.07	19.03	19.20	19.20	19.43	19.44	19.56	19.61
50	*MIRCO-LC*	17.72	17.81	18.04	17.85	17.94	18.04	18.16	18.23
	MIRPO-H1	17.75	17.85	18.15	18.06	18.24	18.27	18.37	18.39
55	*MIRCO-LC*	16.18	16.19	16.40	16.43	16.61	16.73	16.83	16.91
	MIRPO-H1	16.16	16.35	16.55	16.64	16.84	17.01	17.03	17.12
60	*MIRCO-LC*	14.28	14.48	14.58	14.71	14.96	15.13	15.34	15.47
	MIRPO-H1	14.24	14.33	14.86	14.99	15.31	15.35	15.64	15.65
65	*MIRCO-LC*	12.16	12.37	12.68	12.93	13.13	13.24	13.55	13.71
	MIRPO-H1	12.18	12.47	12.81	12.97	13.43	13.51	13.65	13.84
70	*MIRCO-LC*	10.14	10.33	10.49	10.69	11.08	11.34	11.57	11.84
	MIRPO-H1	10.08	10.32	10.51	10.76	11.15	11.37	11.65	11.87
75	*MIRCO-LC*	8.264	8.551	8.563	8.636	8.854	9.044	9.314	9.686
	MIRPO-H1	8.140	8.477	8.468	8.548	8.821	8.947	9.245	9.593

4% annuity value median point predictions

Approach	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*
MIRCO-LC	0.108	0.156	0.178	0.281	0.429	0.547	0.618	0.465
MIRPO-H1	0.191	0.223	0.233	0.345	0.494	0.595	0.646	0.472

Standard deviations, computed across rows (period 1993-07)

Table 1b. E&W 1961-07 female mortality. Evolving biennial 1993(02)07 life expectancy and 4% annuity median predictions (columns), cross-classified by age 40(05)75 and modelling approach (rows), computation along cohort trajectory. K = 2000 simulations. Topping out constraint $q_{109,t_n+109-x} = 1$.

Gaussian joint modelling *MIRCO-LC* approach, data restricted to ages 1-89.
Gaussian joint modelling *MIRPO-H1* approach, data restricted to ages 20-89.

Age	*Approach*	*1993*	*1995*	*1997*	*1999*	*2001*	*2003*	*2005*	*2007*
40	*MIRCO-LC*	0.0333	0.0341	0.0328	0.0311	0.0321	0.0308	0.0302	0.0283
	MIRPO-H1	0.0378	0.0374	0.0355	0.0343	0.0323	0.0317	0.0308	0.0289
45	*MIRCO-LC*	0.0367	0.0373	0.0363	0.0352	0.0348	0.0325	0.0343	0.0325
	MIRPO-H1	0.0443	0.0438	0.0422	0.0410	0.0382	0.0365	0.0356	0.0341
50	*MIRCO-LC*	0.0396	0.0398	0.0370	0.0362	0.0368	0.0361	0.0357	0.0336
	MIRPO-H1	0.0498	0.0491	0.0452	0.0447	0.0424	0.0414	0.0398	0.0379
55	*MIRCO-LC*	0.0444	0.0445	0.0429	0.0409	0.0399	0.0385	0.3889	0.0378
	MIRPO-H1	0.0631	0.0623	0.0591	0.0565	0.0526	0.0505	0.0495	0.0480
60	*MIRCO-LC*	0.0469	0.0471	0.0467	0.0431	0.0426	0.0406	0.0402	0.0374
	MIRPO-H1	0.0728	0.0711	0.0672	0.0649	0.0612	0.0579	0.0562	0.0523
65	*MIRCO-LC*	0.0517	0.0516	0.0496	0.0478	0.0466	0.0447	0.0445	0.0429
	MIRPO-H1	0.0892	0.0873	0.0825	0.0786	0.0739	0.0699	0.0682	0.0651
70	*MIRCO-LC*	0.0588	0.0596	0.0573	0.0551	0.0546	0.0514	0.0512	0.0485
	MIRPO-H1	0.1104	0.1090	0.1034	0.0990	0.0931	0.0886	0.0853	0.0805
75	*MIRCO-LC*	0.0543	0.0553	0.0538	0.0525	0.0532	0.0513	0.0517	0.0502
	MIRPO-H1	0.1192	0.1172	0.1126	0.1087	0.1038	0.0984	0.0952	0.0902

Relative distance of the 99.5% quantile from the median, male mortality, 4% annuity

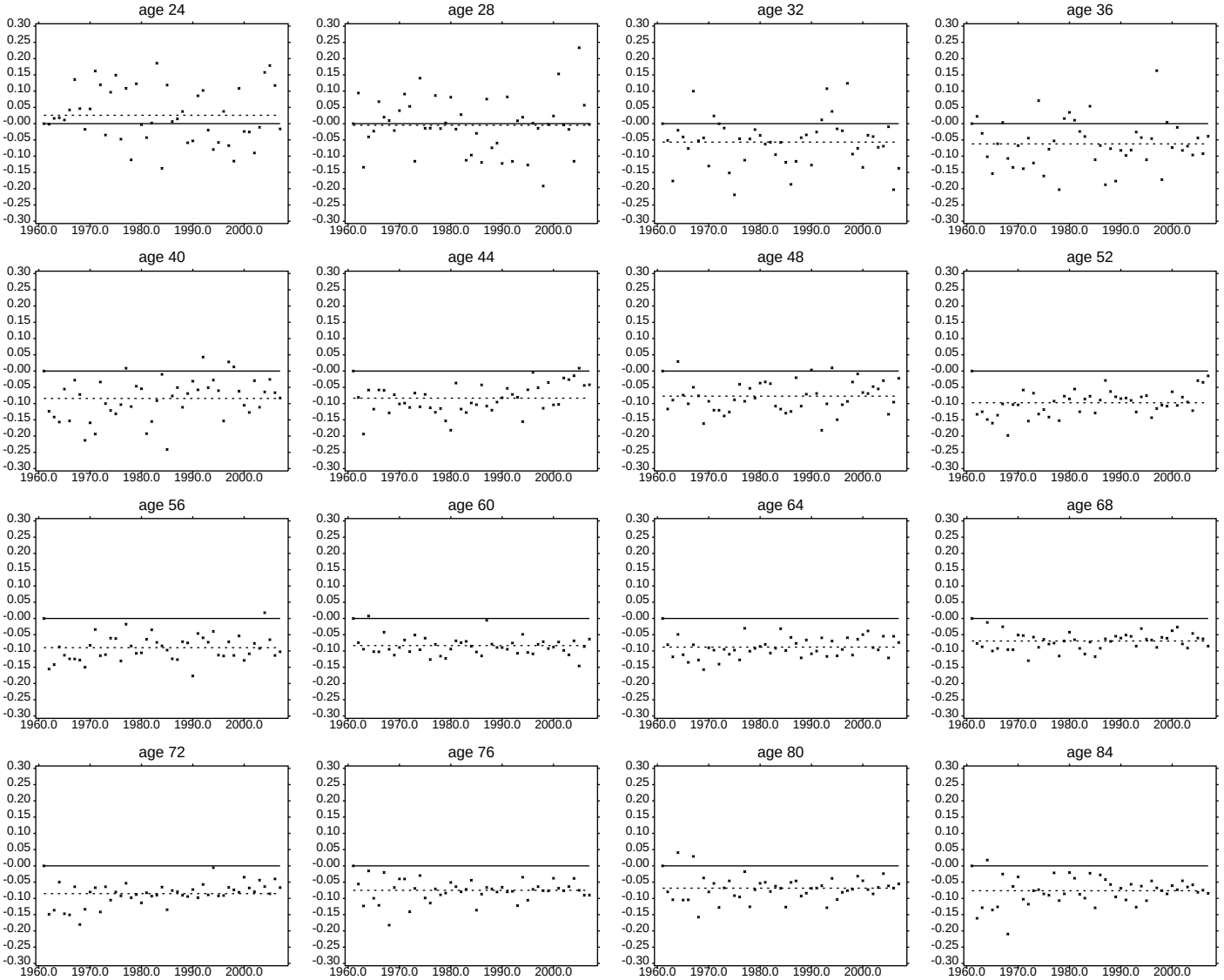
Age	*Approach*	*1993*	*1995*	*1997*	*1999*	*2001*	*2003*	*2005*	*2007*
40	*MIRCO-LC*	0.0234	0.0236	0.0226	0.0218	0.0207	0.0211	0.0207	0.0202
	MIRPO-H1	0.0281	0.0277	0.0259	0.0247	0.0227	0.0225	0.0219	0.0220
45	*MIRCO-LC*	0.0272	0.278	0.0261	0.0262	0.0255	0.0249	0.0238	0.0236
	MIRPO-H1	0.0339	0.0341	0.0320	0.0313	0.0287	0.0281	0.0273	0.0268
50	*MIRCO-LC*	0.0301	0.0300	0.0276	0.0279	0.0275	0.0261	0.0256	0.0248
	MIRPO-H1	0.0379	0.0372	0.0341	0.0339	0.0320	0.0312	0.0306	0.0301
55	*MIRCO-LC*	0.0389	0.0394	0.0379	0.0359	0.0346	0.0329	0.0325	0.0318
	MIRPO-H1	0.0514	0.0498	0.0473	0.0454	0.0429	0.0407	0.0406	0.0395
60	*MIRCO-LC*	0.0440	0.0430	0.0415	0.0398	0.0384	0.0368	0.0358	0.0346
	MIRPO-H1	0.0600	0.0578	0.0548	0.0529	0.0501	0.0476	0.0473	0.0451
65	*MIRCO-LC*	0.0543	0.0541	0.0508	0.0480	0.0467	0.0452	0.0435	0.0421
	MIRPO-H1	0.0735	0.0712	0.0672	0.0643	0.0596	0.0578	0.0568	0.0545
70	*MIRCO-LC*	0.0647	0.0647	0.0619	0.0591	0.0566	0.0541	0.0527	0.0503
	MIRPO-H1	0.0918	0.0898	0.0865	0.0824	0.0781	0.0746	0.0725	0.0697
75	*MIRCO-LC*	0.0646	0.0641	0.0625	0.0606	0.0593	0.0574	0.0560	0.0535
	MIRPO-H1	0.0994	0.0959	0.0935	0.0907	0.0872	0.0843	0.0822	0.0780

Relative distance of the 99.5% quantile from the median, female mortality, 4% annuity

Table 2. E&W 1961-07 male (upper) and female (lower) mortality experiences. Relative distance of the predicted 1 in 200 upper quantiles from the respective median predictions (columns), for evolving 1993(02)07 period 4% annuities: cross-classified by age 40(05)75 and modelling approach (rows), computation along cohort trajectory. K = 2000 simulations. Topping out constraint $q_{109,t_n+109-x} = 1$.

Gaussian joint modelling *MIRCO-LC* approach, data restricted to ages 1-89.
Gaussian joint modelling *MIRPO-H1* approach, data restricted to ages 20-89.

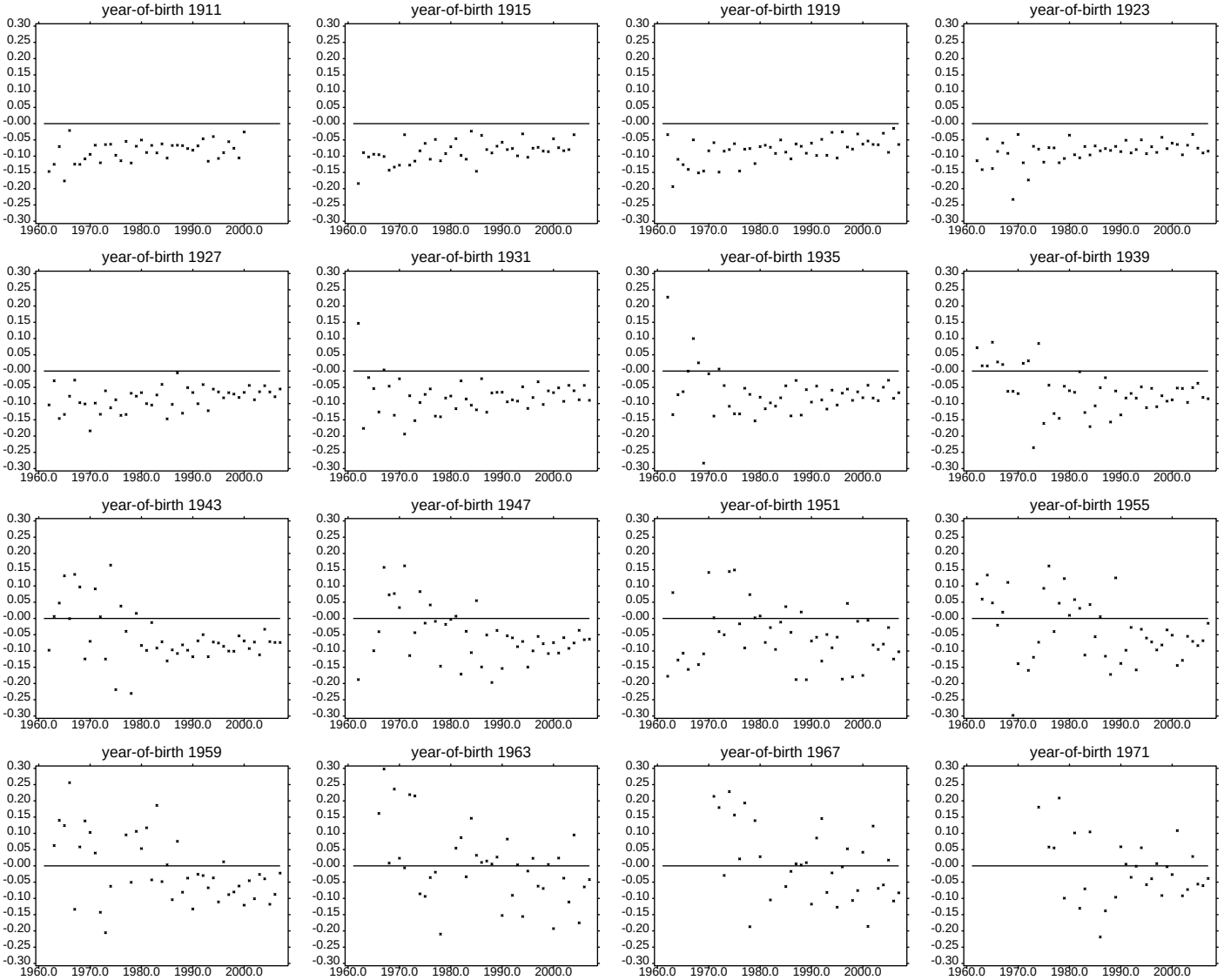
Figure



line), plotted against calendar year for individual ages 24(04)80.

Fig 1a. England & Wales 1961-2007 male mortality experience. Mortality improvement rates by cohort orientation (MIRCO) with mean (dotted

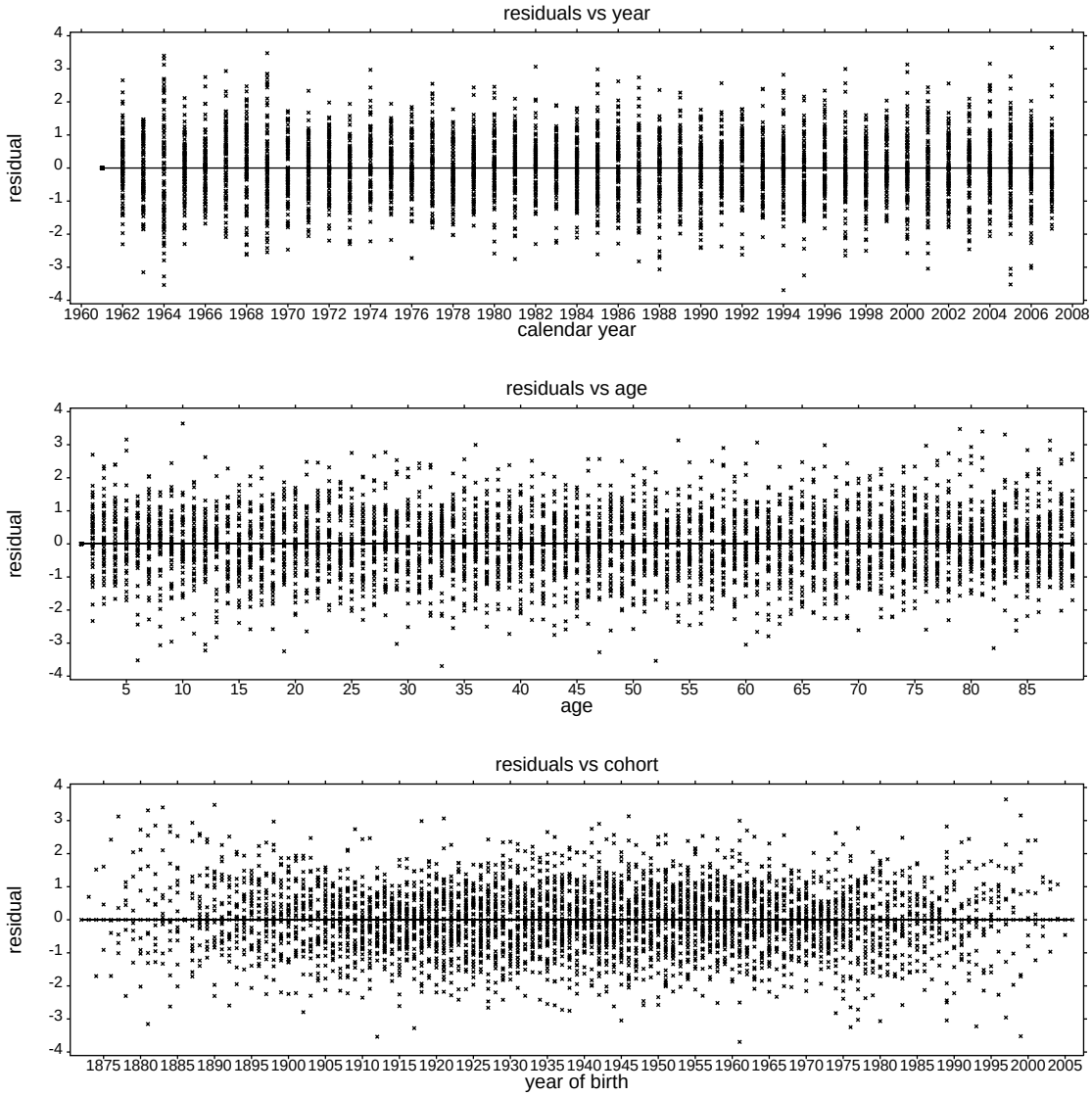
Figure



of-observation for individual years-of-birth 1911(04)71.

Fig 1b. England & Wales 1961-2007 male mortality experience. Mortality improvement rates by cohort orientation (MIRCO) plotted against year-

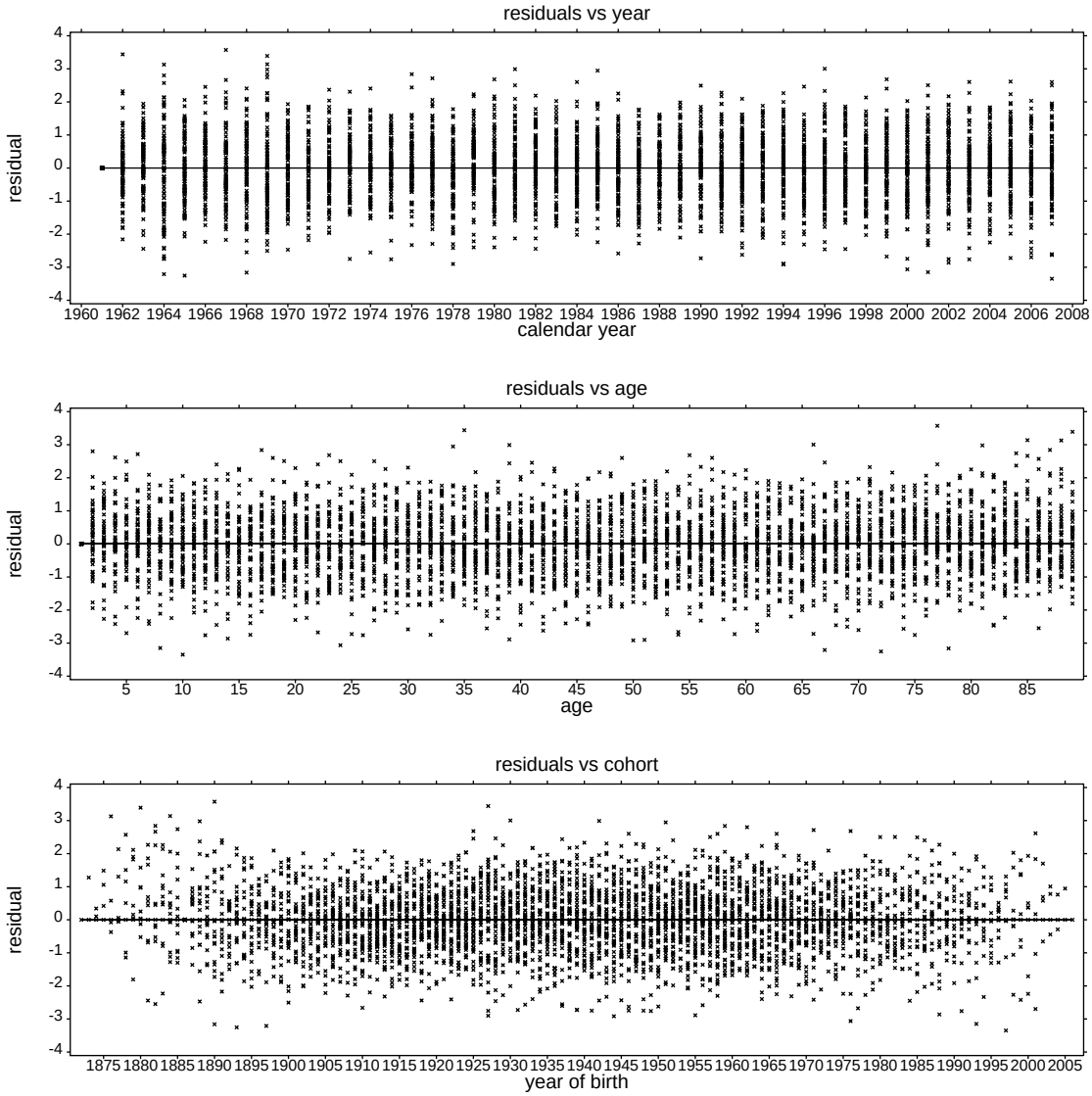
Figure



Scaled deviance residual plots against (a) period, (b) age, (c) cohort.

Fig 2a. England & Wales 1961-2007 male mortality experience, ages 1-89. Gaussian MIRCO-LC first stage response model, fitted by joint modelling.

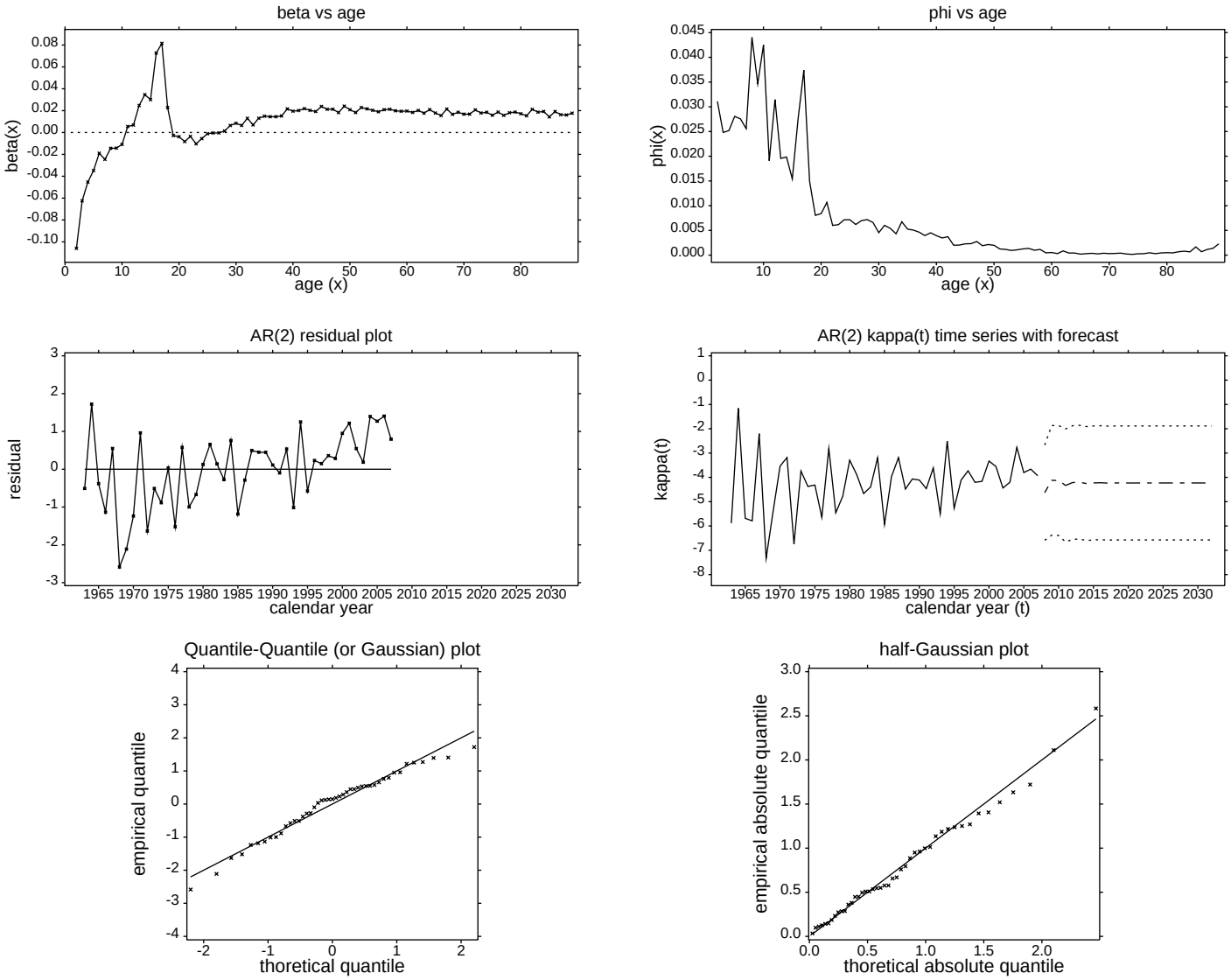
Figure



Scaled deviance residual plots against (a) period, (b) age, (c) cohort.

Fig 2b. England & Wales 1961-2007 female mortality experience, ages 1-89. Gaussian MIRCO-LC first stage response model, fitted by joint modelling.

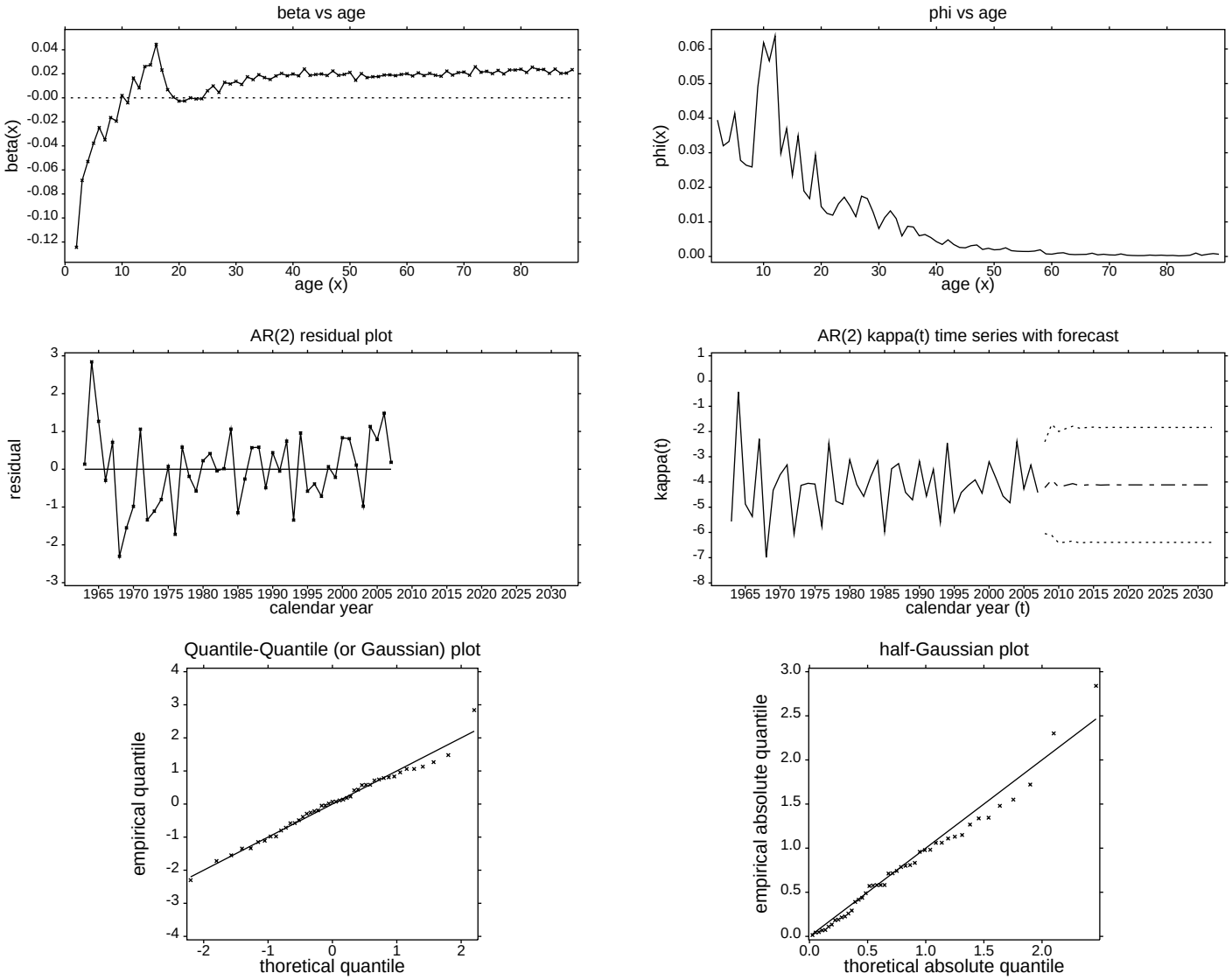
Figure



Upper panels: respective beta and phi estimates. Centre panels: AR(2) period component process, residuals (left) and time series with forecast. Lower panels: Q-Q Gaussian and half-Gaussian AR(2) residual plots.

Fig 3a. England & Wales 1961-2007 male mortality experience, ages 1-89. Gaussian MIRCO-LC first stage response model, fitted by joint modelling.

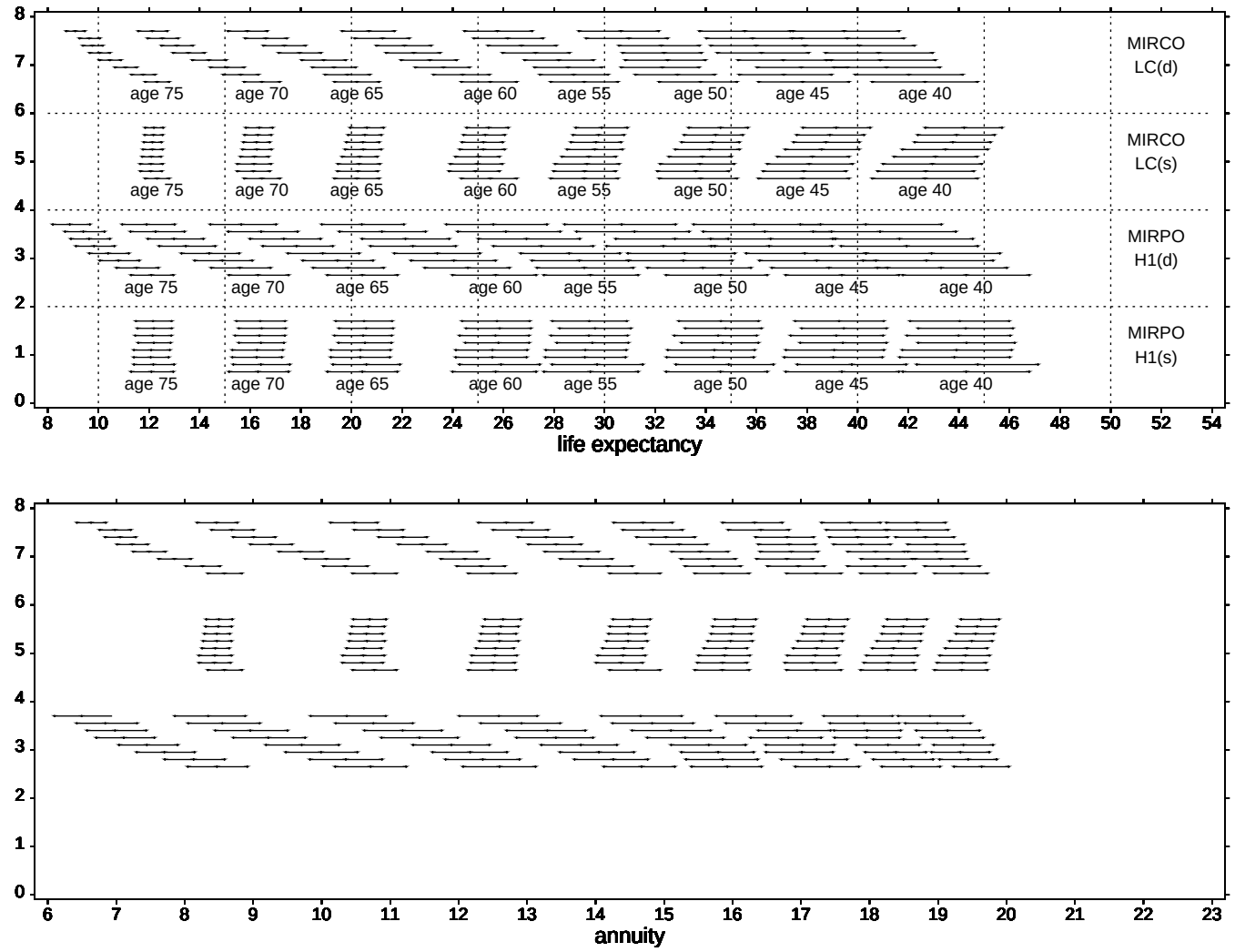
Figure



Upper panels: respective beta and phi estimates. Centre panels: AR(2) period component process, residuals (left) and time series with forecast. Lower panels: Q-Q Gaussian and half-Gaussian AR(2) residual plots.

Fig 3b. England & Wales 1961-2007 female mortality experience, ages 1-89. Gaussian MIRCO-LC first stage response model, fitted by joint modelling.

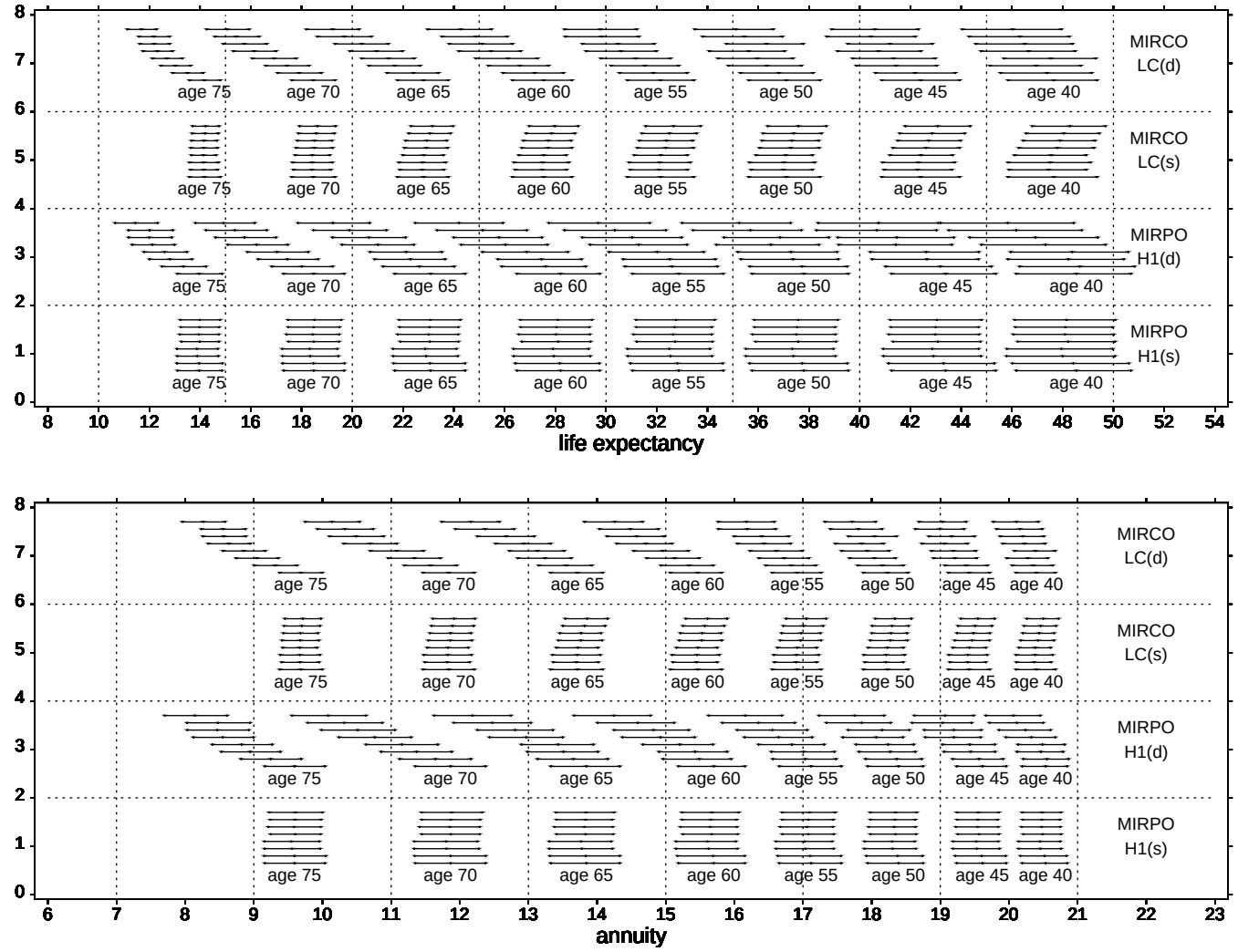
Figure



1993(02)07 dynamic (d) predictions (decending sequence) juxtaposed with static (s) 2007 predictions, subject to biennial 1961(02)75 data deletions (ascending sequence), ages 40(05)75. Joint model fitting. Comparing:- Gaussian MIRCO approach: LC structure, data age range 1-89. Gaussian MIRPO approach: H1 structure, data age range 20-89.

Fig 4a. E&W 1961-2007 male mortality. Life expectancy and 4% annuity 5%, 50%, 95% quantile predictions. K=2000 simulations. Evolving biennial

Figure



1993(02)07 dynamic (d) predictions (decending sequence) juxtaposed with static (s) 2007 predictions, subject to biennial 1961(02)75 data deletions (ascending sequence), ages 40(05)75. Joint model fitting. Comparing:- Gaussian MIRCO approach: LC structure, data age range 1-89. Gaussian MIRPO approach: H1 structure, data age range 20-89.

Fig 4b. E&W 1961-2007 female mortality. Life expectancy and 4% annuity 5%, 50%, 95% quantile predictions. K=2000 simulations. Evolving biennial

Figure

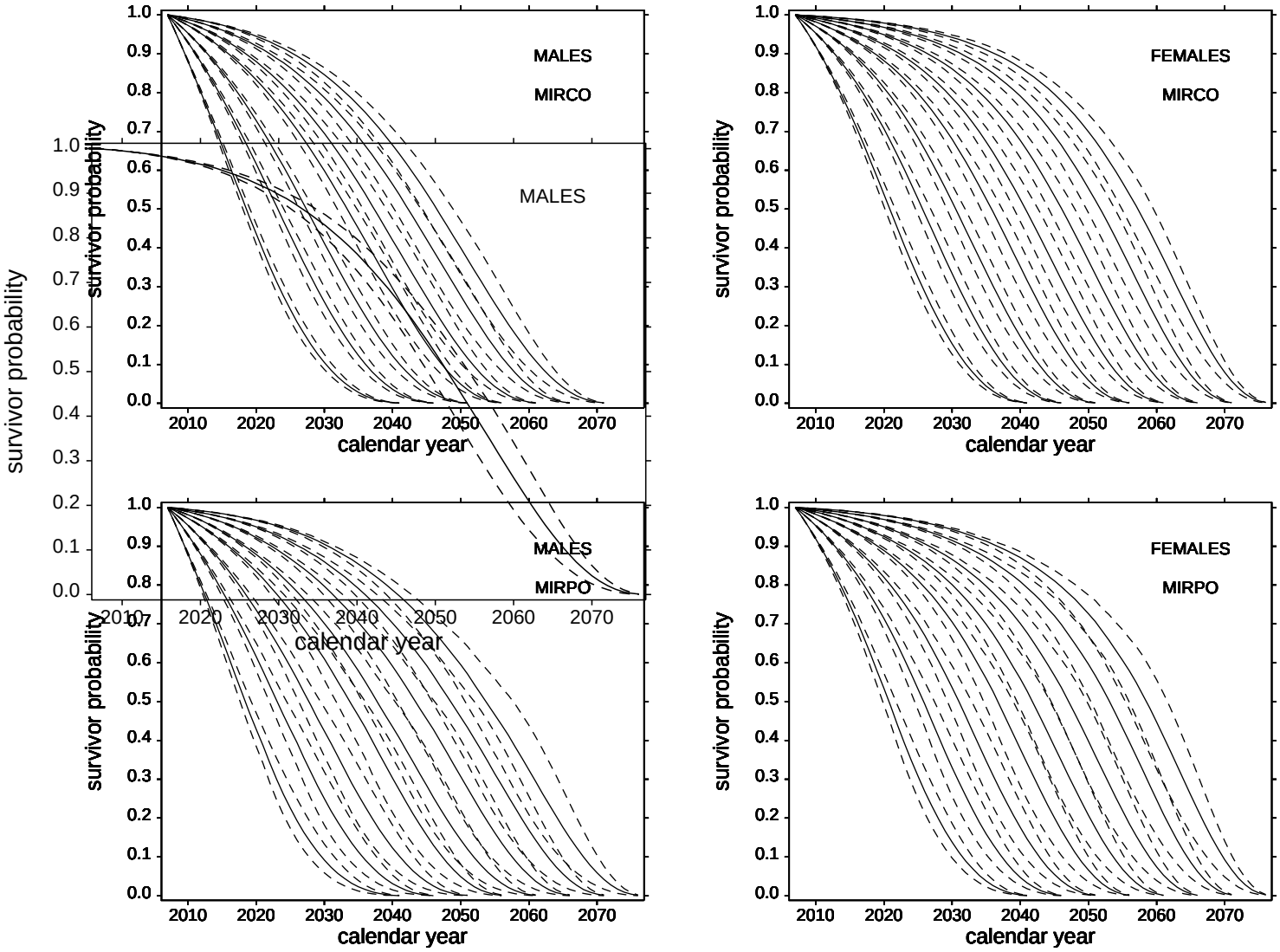
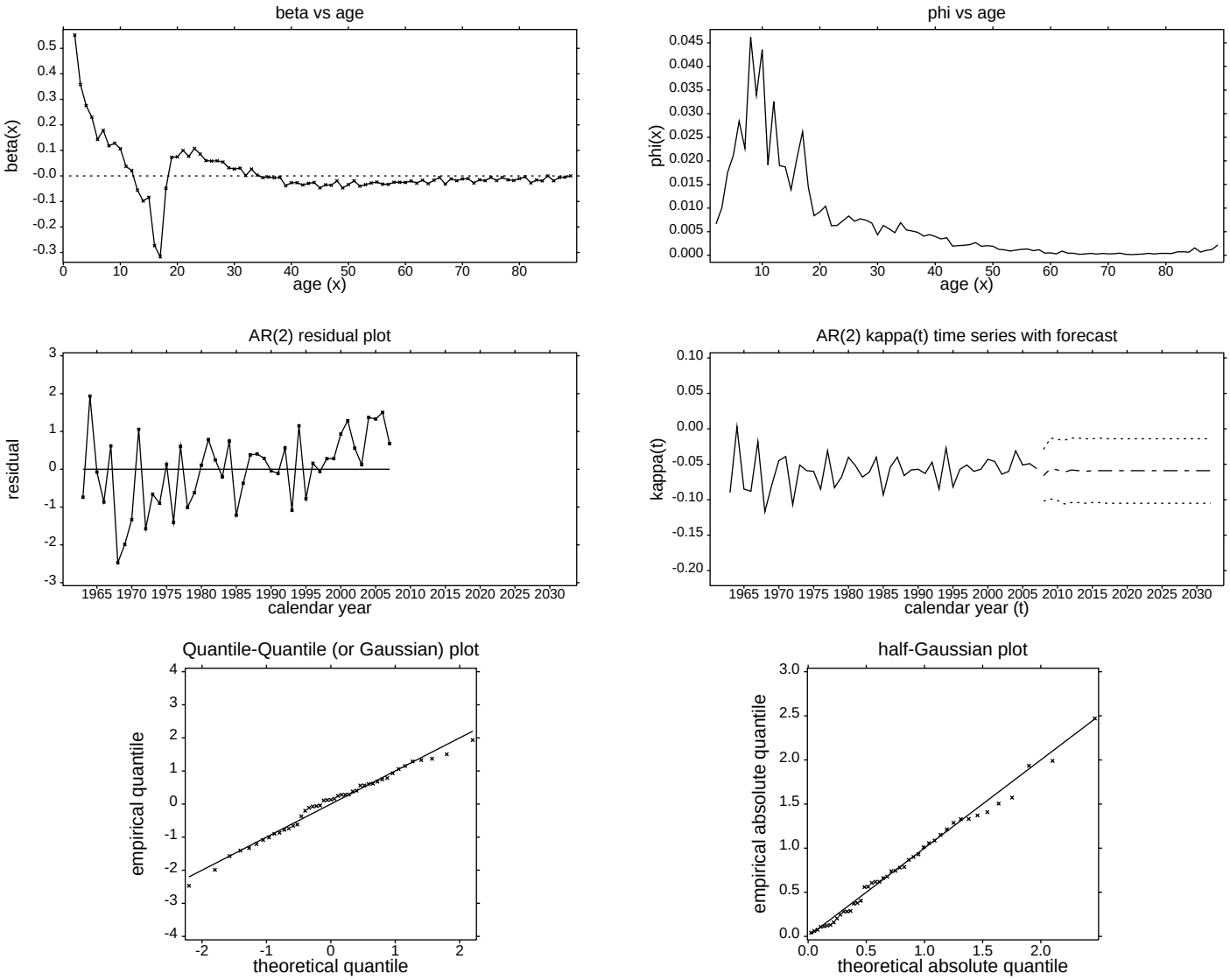


Fig 5. E&W 1961-2007 mortality. Survivor probability function 5%,50%,95% quantiles. K=2000 simulations. Period 2007 predictions for ages 40(05)75 (right to left). Joint model fitting. Comparing:-
Gaussian MIRCO approach: LC structure, age range 1-89 (upper panels)
Gaussian MIRPO approach: H1 structure, age range 20-89 (lower panels).
Male experiece LH panels, female experience RH panels.

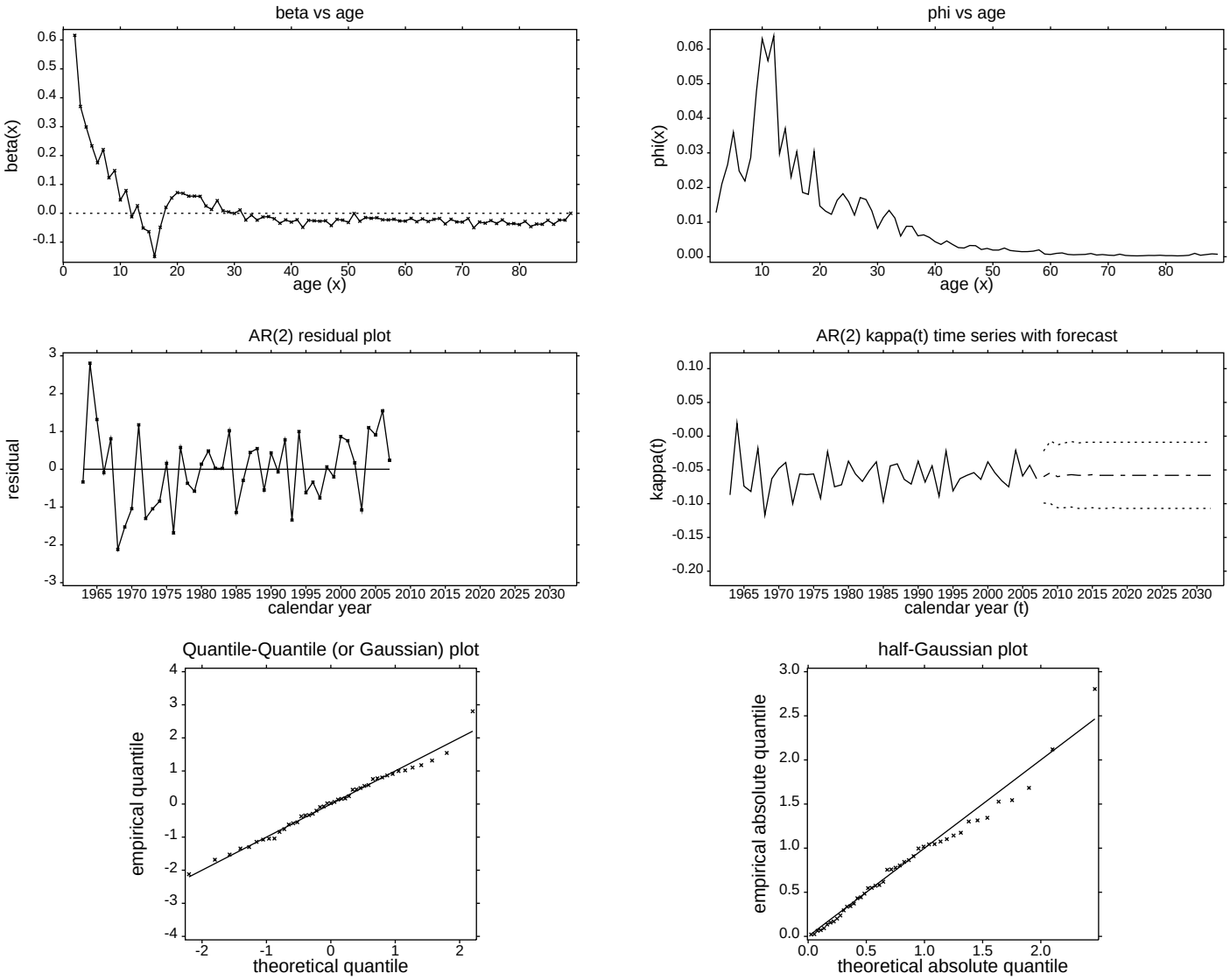
Figure



Upper panels: respective beta and phi estimates. Centre panels: AR(2) period component process, residuals (left) and time series with forecast. Lower panels: Q-Q Gaussian and half-Gaussian AR(2) residual plots.

Fig 6a. England & Wales 1961-2007 male mortality experience, ages 1-89. Gaussian MIRCO-H2 first stage response model, fitted by joint modelling.

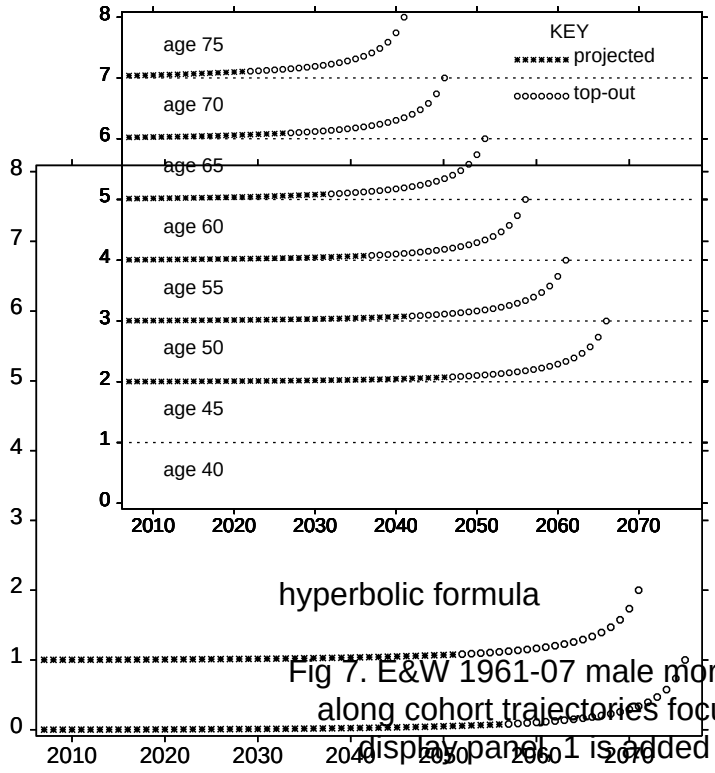
Figure



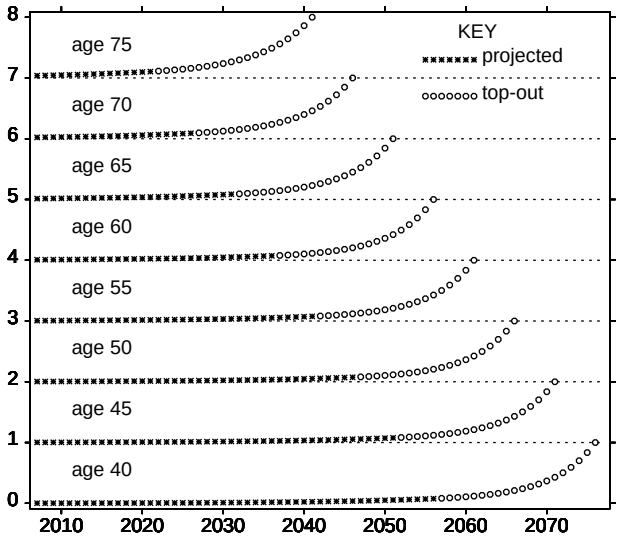
Upper panels: respective beta and phi estimates. Centre panels: AR(2) period component process, residuals (left) and time series with forecast. Lower panels: Q-Q Gaussian and half-Gaussian AR(2) residual plots.

Fig 6b. England & Wales 1961-2007 female mortality experience, ages 1-89. Gaussian MIRCO-H2 first stage response model, fitted by joint modelling.

Figure



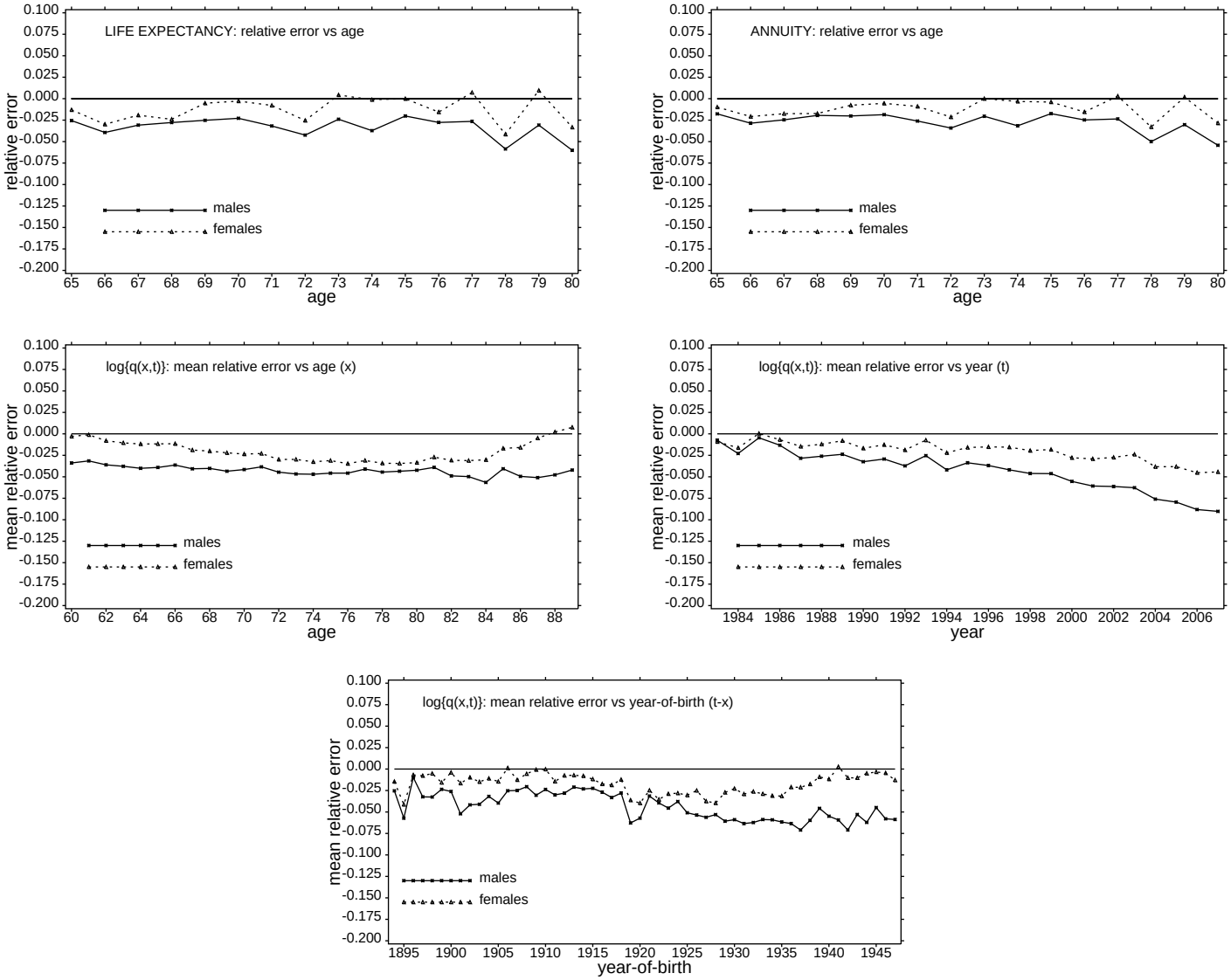
hyperbolic formula



log- quadratic formula

Fig 7. E&W 1961-07 male mortality. Projected and top-out mortality rates along cohort trajectories focused on 2007 for ages 40(05)75. [In each display panel, 1 is added to the rates, for each increase in age]. Gaussian MIRCO approach: LC structure, data age range 1-89.

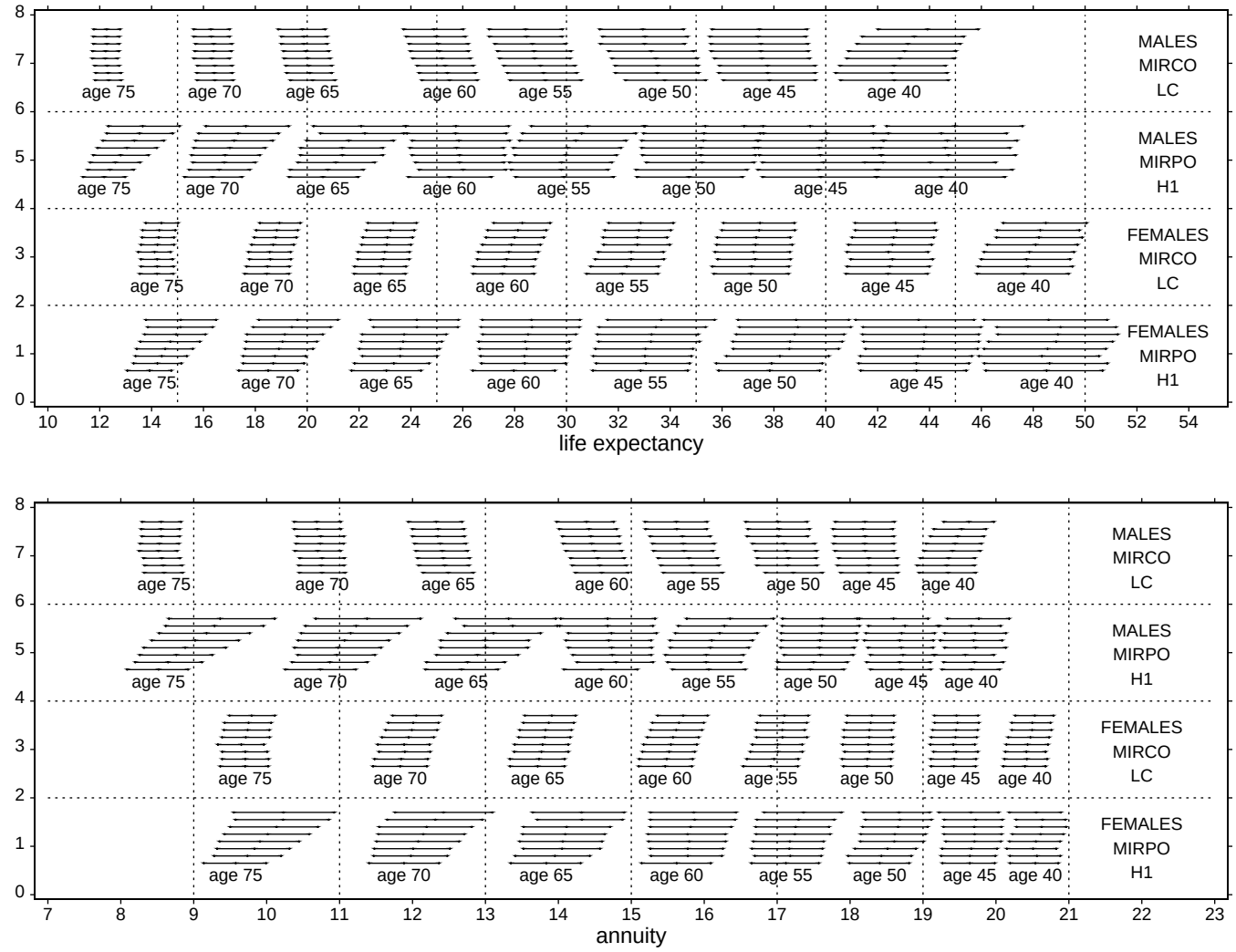
Figure



values for ages 65-80 (upper panels). Retrospective relative error in the 1982 predicted log death rates, averaged over ages, years, cohorts respectively (centre and lower panels). Averages based on rectangular region bounded by the ages 60-89 and period 1983-2007.

Fig 8. E&W male and female mortality experiences. Retrospective relative error in the 1982 predicted life expectancy and 4% annuity

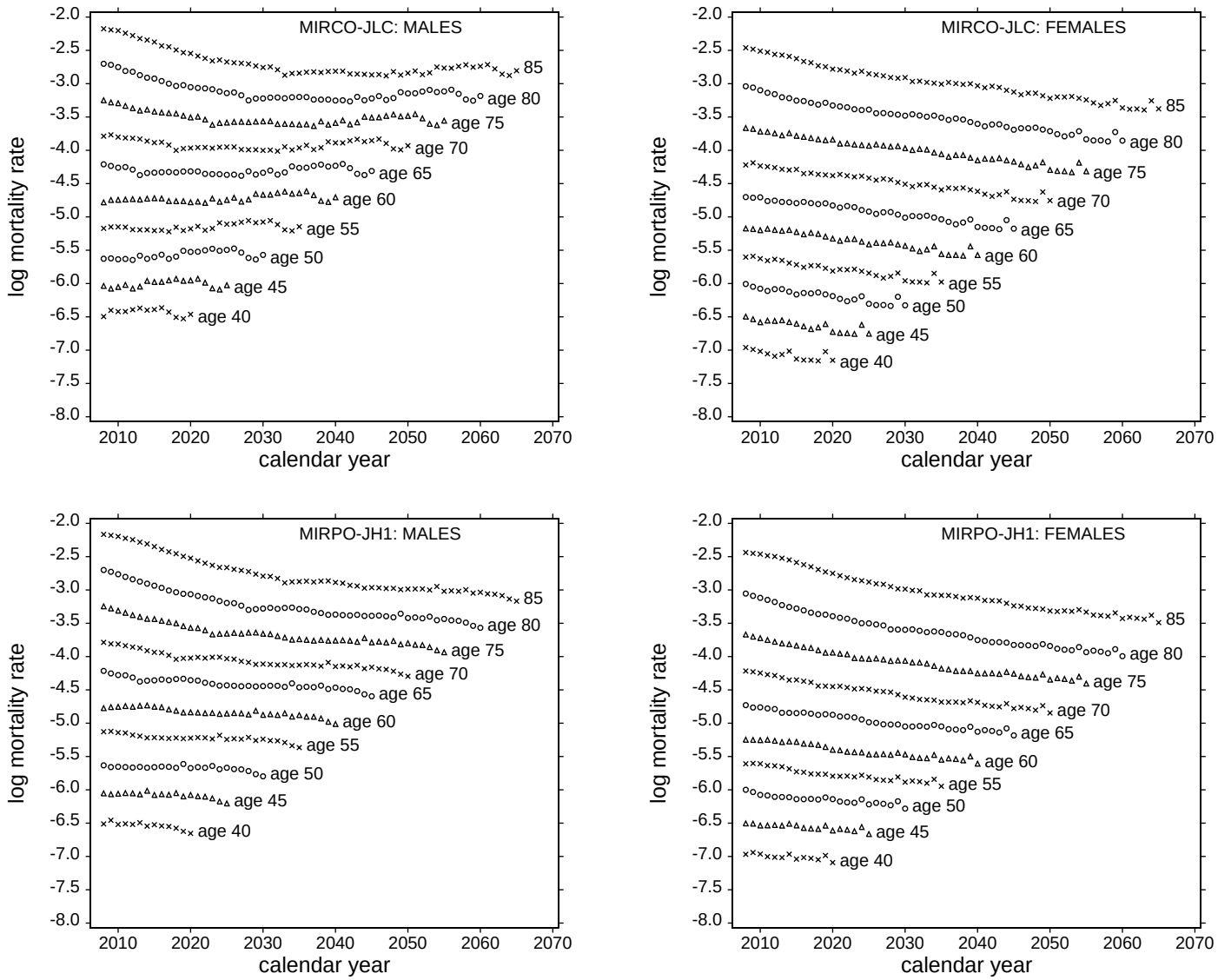
Figure



forward 2007(01)14 focused predictions (ascending sequence), ages 40(05)75. Comparing:- Gaussian MIRCO-LC approached (data age range 1-89) juxtaposed with MIRPO-H1 approach (data age range 20-89). Males bands 1 & 2, females bands 3 & 4. Joint model fitting, full period 1961-07. Computations by cohort trajectory.

Fig 9. E&W 1961-2007 male & female mortality. Life expectancy and 4% annuity 5%, 50%, 95% quantile predictions. K=2000 simulations. Annual

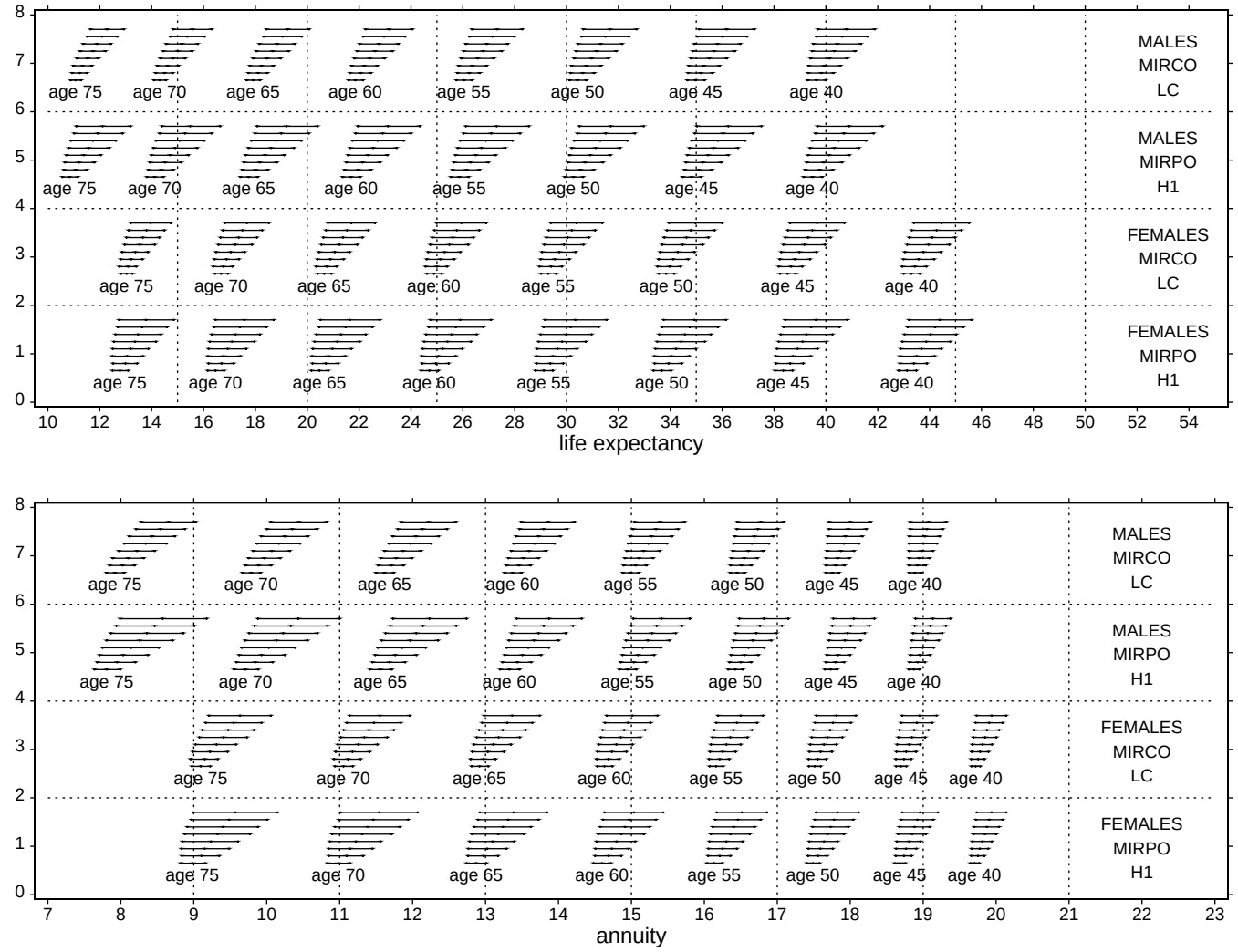
Figure



Upper panels: MIRCO-JLC approach (data age range 1-89).
Lower panels: MIRPO-JH1 approach (data age range 20-89).

Fig 10. E&W 1961-2007 male & female mortality experiences. Log mortality rate predictions, ages 40(05)85. K=2000 simulations.

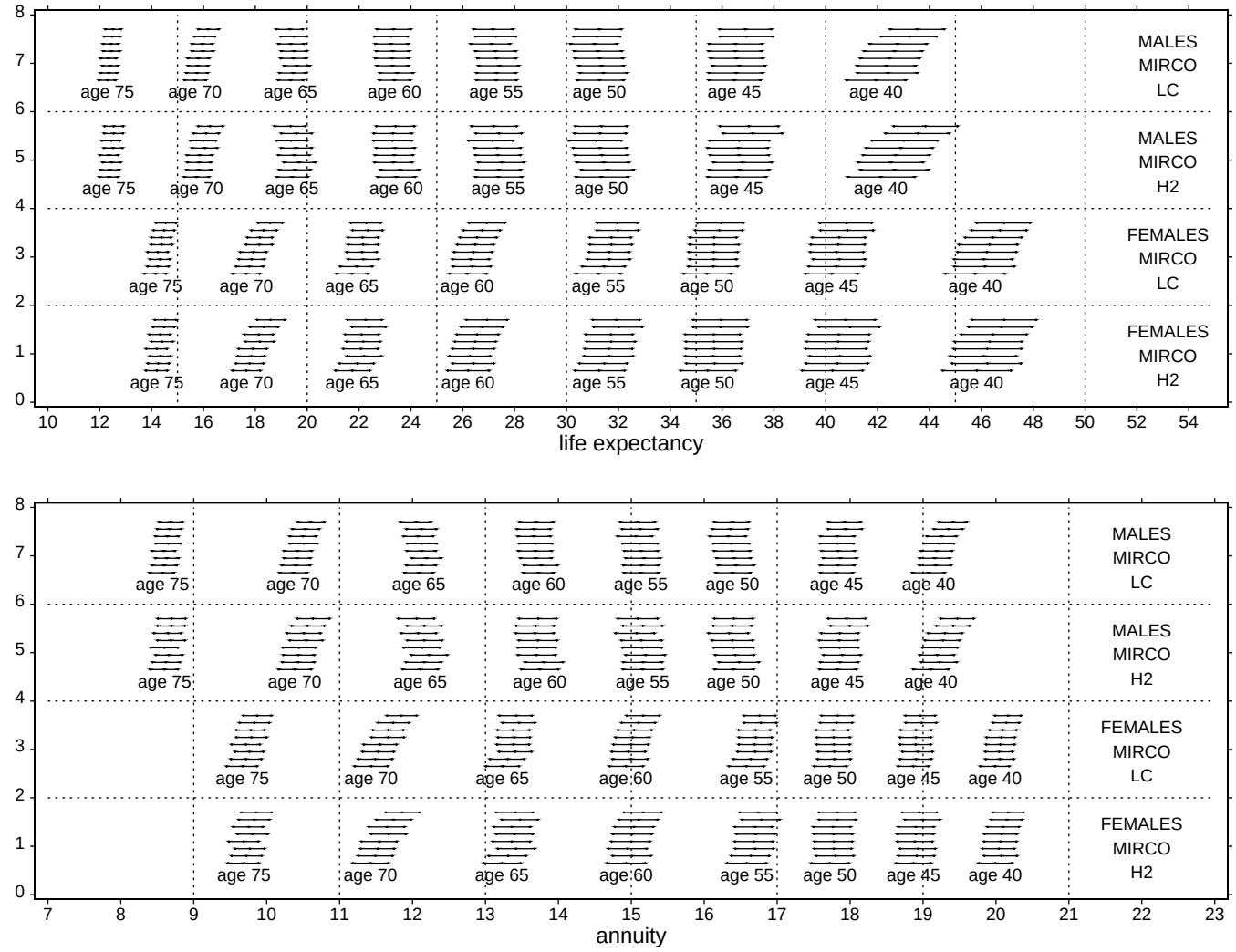
Figure



forward 2008(01)15 focused predictions (ascending sequence), ages 40(05)75. Comparing:- Gaussian MIRCO-LC approached (data age range 1-89) juxtaposed with MIRPO-H1 approach (data age range 20-89). Males bands 1 & 2, females bands 3 & 4. Joint model fitting, full period 1961-07. Computations by period trajectory.

Fig 11. E&W 1961-2007 male & female mortality. Life expectancy and 4% annuity 5%, 50%, 95% quantile predictions. K=2000 simulations. Annual

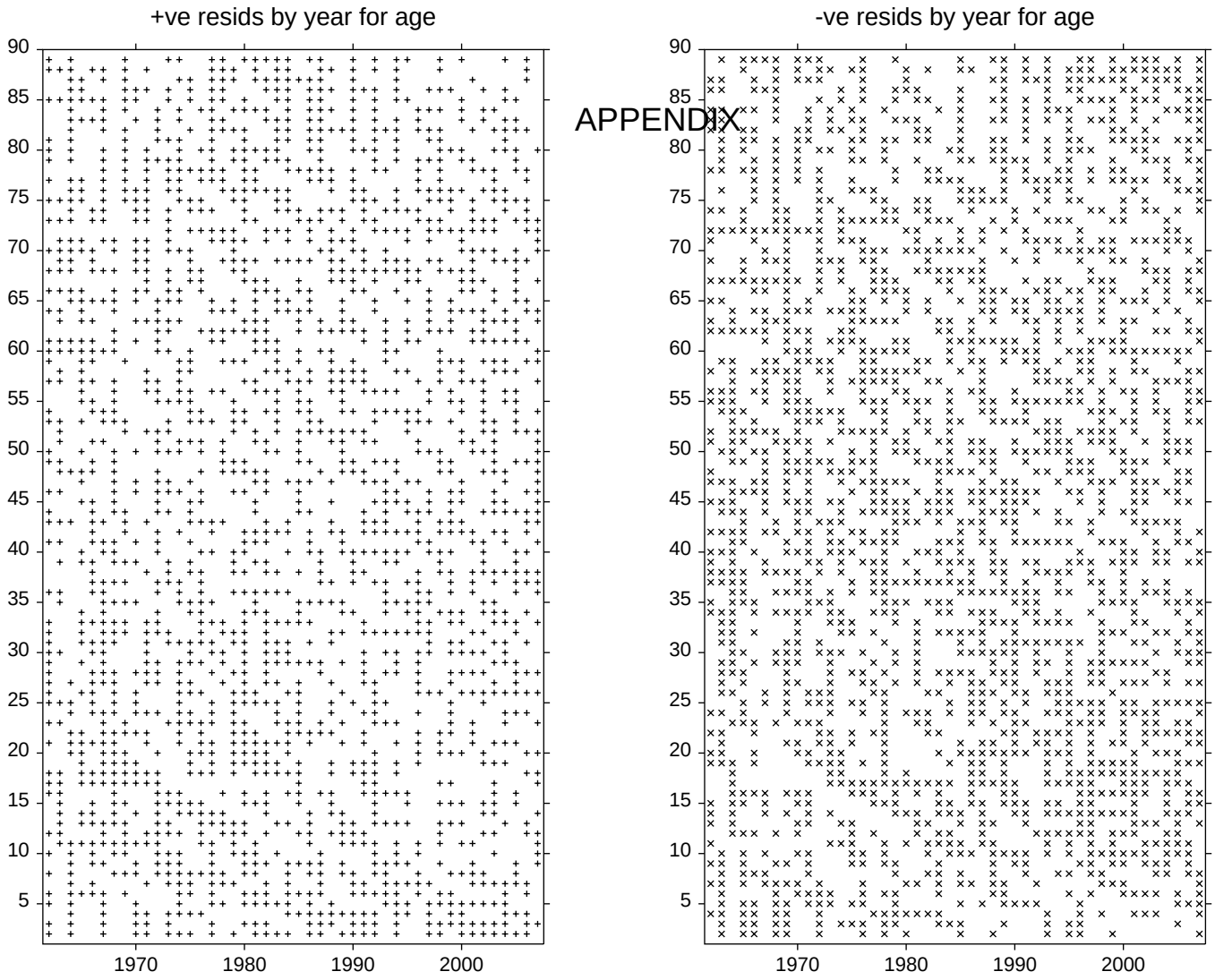
Figure



Annual forward 2006(01)13 focused predictions (ascending sequence), ages 40(05)75. Comparing:- Gaussian MIRCO-LC approached juxtaposed with MIRCO-H2 approach. Males bands 1 & 2, females bands 3 & 4.
Joint model fitting, period 1961-2006, ages 1-89.
Computations by cohort trajectory.

Fig 12. USA 1961-2006 male & female mortality. Life expectancy and 4% annuity 5%, 50%, 95% quantile predictions. K=2000 simulations.

Figure

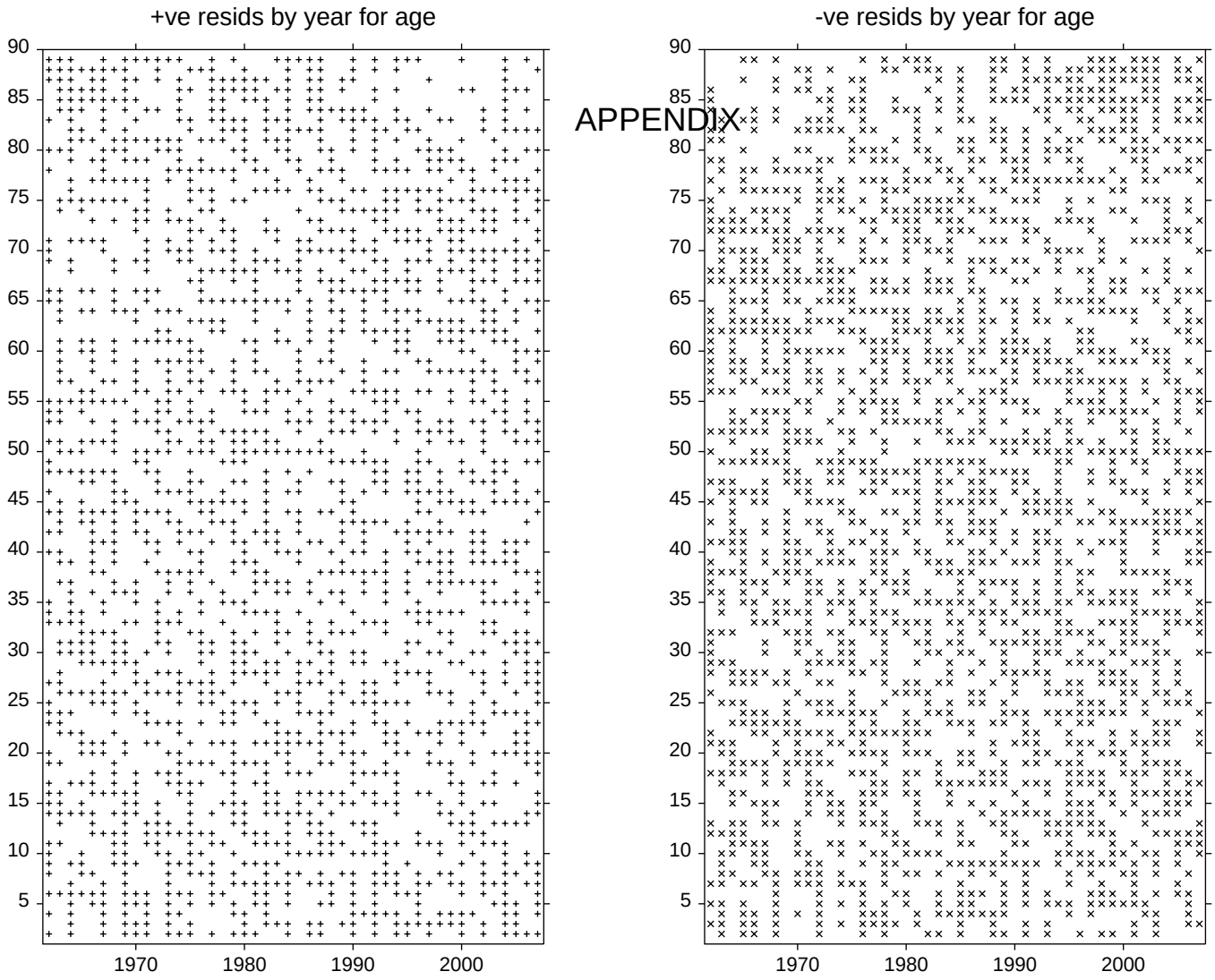


APPENDIX

Gaussian MIRCO-LC first stage response model, fitted by joint modelling.
Deviance residual plots: left panel- positives; right panel- negatives.

Fig A1a. England & Wales 1961-2007 male mortality experience, ages 1-89.

Figure

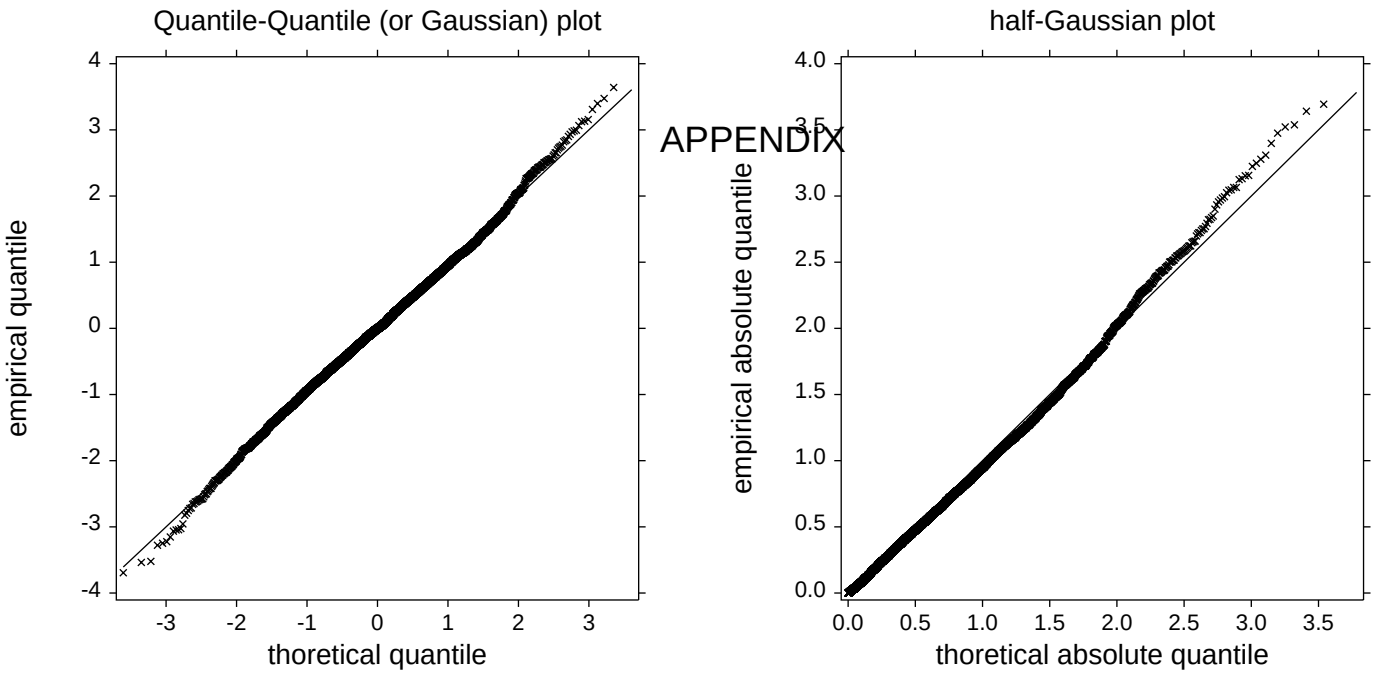


APPENDIX

Gaussian MIRCO-LC first stage response model, fitted by joint modelling.
Deviance residual plots: left panel- positives; right panel- negatives.

Fig A1b. England & Wales 1961-2007 female mortality experience, ages 1-89.

Figure

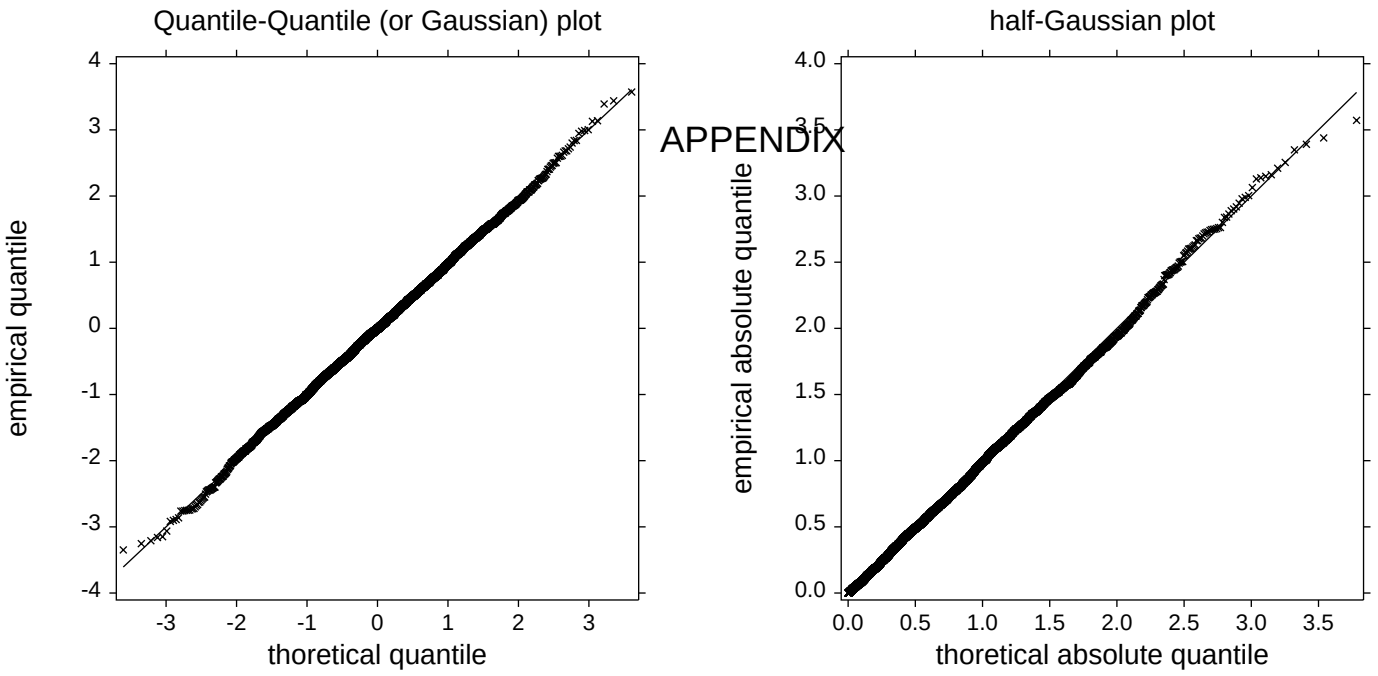


APPENDIX

Gaussian MIRCO-LC first stage response model, fitted by joint modelling.
Ordered deviance residual plots: left panel:- Quantile-Quantile plot
right panel:- half Gaussian plot.

Fig A2a. England & Wales 1961-2007 male mortality experience, ages 1-89.

Figure



APPENDIX

Gaussian MIRCO-LC first stage response model, fitted by joint modelling.
Ordered deviance residual plots: left panel:- Quantile-Quantile plot
right panel:- half Gaussian plot.

Fig A2b. England & Wales 1961-2007 female mortality experience, ages 1-89.

Highlights

In this study, we consider how to model and project mortality rates. We take a different view from the literature. We consider the mortality improvement rates for cohorts and so focus on the underlying trend. This is an alternative to the conventional approach which models directly the mortality rates over time. We set up the modelling framework and then compare results from the different approaches using a case study.

***Revision Checklist**

We have re-drafted the paper and removed all of the bullet point lists except for the final list in section 4.7, where we feel the presentation is appropriate for a listing of key points emerging from the research. We hope that this is acceptable.

We have added an extra sentence in section 3.6 about the approved approximate approach to the Solvency II SCR calculation. Also, in section 3.6, we have strengthened the comments about the limited perspective that we have taken on the SCR and the limited relevance of our calculations. We have decided not to undertake a full Solvency II set of simulations because this would change the objectives of the paper - indeed, this could be a topic for a separate piece of work.